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## Surds

### Things to remember:

- $\sqrt{\quad}$  means square root;
- To simplify surds, find all its factors;
- To rationalise the denominator, find an equivalent fraction where the denominator is rational.

### Questions:

1. Work out

$$\frac{(5 + \sqrt{3})(5 - \sqrt{3})}{\sqrt{22}}$$

Give your answer in its simplest form.

$$\frac{(25 - \cancel{5\sqrt{3}} + \cancel{5\sqrt{3}} - 3)\sqrt{22}}{22}$$
$$= \frac{22\sqrt{22}}{22}$$

..... $\sqrt{22}$ .....  
(Total 3 marks)

2. (a) Rationalise the denominator of  $\frac{1}{\sqrt{3}}$

..... $\frac{\sqrt{3}}{3}$ .....  
(1)

(b) Expand  $(2 + \sqrt{3})(1 + \sqrt{3})$

Give your answer in the form  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers.

$$2 + 2\sqrt{3} + \sqrt{3} + 3$$

..... $5 + 3\sqrt{3}$ .....  
(2)  
(Total 3 marks)

3. (a) Rationalise the denominator of  $\frac{1}{\sqrt{7}}$

$$\frac{\sqrt{7}}{7}$$

.....

(2)

- (b) (i) Expand and simplify  $(\sqrt{3} + \sqrt{15})^2$   
Give your answer in the form  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers.

$$\begin{aligned} & 3 + \sqrt{45} + \sqrt{45} + 15 \\ & = 18 + 2\sqrt{45} \end{aligned}$$

.....  $18 + 6\sqrt{5}$  .....

- (ii) All measurements on the triangle are in centimetres.  
 $ABC$  is a right-angled triangle.  
 $k$  is a positive integer.

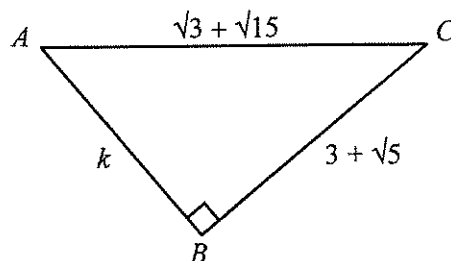


Diagram NOT accurately drawn

Find the value of  $k$ .

$$\begin{aligned} & (\sqrt{3} + \sqrt{15})(\sqrt{3} + \sqrt{15}) - (3 + \sqrt{5})(3 + \sqrt{5}) \\ & = (18 + 6\sqrt{5}) - (14 + 6\sqrt{5}) \end{aligned}$$

$k =$  .....  $4$  .....

(5)

(Total 7 marks)

4. Expand and simplify  $(\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})$

$$3 - \sqrt{6} - \sqrt{6} + 2$$

.....  $5 - 2\sqrt{6}$  .....  
(Total 2 marks)

5. (a) Write down the value of  $49^{1/2}$   $\sqrt{49}$

..... 7 .....  
(1)

(b) Write  $\sqrt{45}$  in the form  $k\sqrt{5}$ , where  $k$  is an integer.

$$\sqrt{9} \times \sqrt{5}$$

.....  $3\sqrt{5}$  .....  
(1)  
(Total 2 marks)

6. Write  $\frac{\sqrt{18} + 10}{\sqrt{2}}$  in the form  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers.

$$= \frac{\sqrt{36} + 10\sqrt{2}}{2}$$

$$= \frac{6 + 10\sqrt{2}}{2}$$

$$= 3 + 5\sqrt{2}$$

$a =$  ..... 3 .....

$b =$  ..... 5 .....

(Total 2 marks)

7. Expand and simplify  $(2 + \sqrt{3})(7 - \sqrt{3})$   
 Give your answer in the form  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers.

$$14 - 2\sqrt{3} + 7\sqrt{3} - 3$$

$$\dots\dots\dots 11 + 5\sqrt{3} \dots\dots\dots$$

(Total 3 marks)

8. Rationalise the denominator of  $\frac{(4 + \sqrt{2})(4 - \sqrt{2})}{\sqrt{7}}$   
 Give your answer in its simplest form.

$$\frac{(16 + 4\cancel{\sqrt{2}} - 4\cancel{\sqrt{2}} - 2)\sqrt{7}}{7}$$

$$= \frac{14\sqrt{7}}{7}$$

$$\dots\dots\dots 2\sqrt{7} \dots\dots\dots$$

(Total for question = 3 marks)

9. Show that  $\frac{(4 - \sqrt{3})(4 + \sqrt{3})}{\sqrt{13}}$  simplifies to  $\sqrt{13}$

$$= \frac{(16 - 4\cancel{\sqrt{3}} + 4\cancel{\sqrt{3}} - 3)\sqrt{13}}{13}$$

$$= \frac{13\sqrt{13}}{13}$$

(Total for question = 2 marks)

## Bounds Calculations

### Things to remember:

- Calculating bounds is the opposite of rounding – they are the limits at which you would round up instead of down, and vice versa.
- When dividing bounds,  $UB = UB \div LB$  and  $LB = LB \div UB$

### Questions:

1. A piece of wood has a length of 65 centimetres to the nearest centimetre.

(a) What is the least possible length of the piece of wood?

..... 64.5 ..... (1)

(b) What is the greatest possible length of the piece of wood?

..... 65.5 ..... (1)

(Total for Question is 2 marks)

2. Chelsea's height is 168 cm to the nearest cm.

(a) What is Chelsea's minimum possible height?

..... 167.5 ..... cm (1)

(b) What is Chelsea's maximum possible height?

..... 168.5 ..... cm (1)

(Total for Question is 2 marks)

3.  $I = \frac{V}{R}$

$V = 250$  correct to the nearest 5

$R = 3900$  correct to the nearest 100

Work out the lower bound for the value of  $I$ .

Give your answer correct to 3 decimal places.

You must show your working.

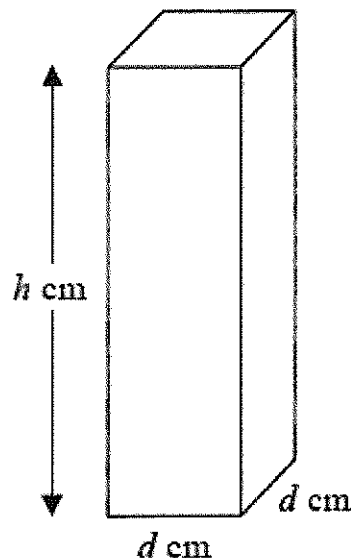
$$LB \text{ of } V = 247.5$$

$$UB \text{ of } R = 3950$$

$$I = \frac{247.5}{3950} = 0.06265\dots$$

..... 0.063 .....  
(Total for question = 3 marks)

4. Here is a solid bar made of metal.  
 The bar is in the shape of a cuboid.  
 The height of the bar is  $h$  cm.  
 The base of the bar is a square of side  $d$  cm.  
 The mass of the bar is  $M$  kg.



$d = 8.3$  correct to 1 decimal place.  
 $M = 13.91$  correct to 2 decimal places.  
 $h = 84$  correct to the nearest whole number.

Find the value of the density of the metal to an appropriate degree of accuracy.  
 Give your answer in  $\text{g/cm}^3$ .  
 You must explain why your answer is to an appropriate degree of accuracy.

$$\begin{aligned} \text{UB} &= \frac{\text{UB}(M)}{\text{LB}(h) \times \text{LB}(d)^2} \\ &= \frac{13.915}{83.5 \times 8.25^2} \\ &= 0.002448436 \end{aligned}$$

$$\begin{aligned} \text{LB} &= \frac{\text{LB}(M)}{\text{UB}(h) \times \text{UB}(d)^2} \\ &= \frac{13.905}{84.5 \times 8.35^2} \\ &= 0.002360159 \end{aligned}$$

Density =  $2.4 \text{ g/cm}^3$  as this is correct for both the upper and lower bounds.

(Total for question = 5 marks)

5. Steve travelled from Ashton to Barnfield.  
 He travelled 235 miles, correct to the nearest 5 miles.  
 The journey took him 200 minutes, correct to the nearest 5 minutes.  
 Calculate the lower bound for the average speed of the journey.  
 Give your answer in **miles per hour**, correct to 3 significant figures.  
 You must show all your working.



$$\text{LB}(S) = \frac{\text{LB}(D)}{\text{UB}(T)} = \frac{232.5}{202.5 \div 60} = 68.888... \text{ mph}$$

..... 68.9 ..... mph  
 (Total for question = 4 marks)

6. The value of  $p$  is 4.3  
 The value of  $q$  is 0.4  
 Both  $p$  and  $q$  are given correct to the nearest 0.1  
 (a) Write down the lower bound for  $p$ .

..... 4.25 .....  
 (1)

$$r = p + \frac{1}{q}$$

- (b) Work out the upper bound for  $r$ .  
 You must show all your working.

$$\begin{aligned} \text{UB}(r) &= \text{UB}(p) + \frac{1}{\text{LB}(q)} \\ &= 4.35 + \frac{1}{0.35} \\ &= 7.20714\dots \end{aligned}$$

..... 7.21 (2 d.p.) .....  
 (3)

(Total for question = 4 marks)

$$m = \frac{\sqrt{s}}{t}$$

$s = 3.47$  correct to 3 significant figures

$t = 8.132$  correct to 4 significant figures

7.

By considering bounds, work out the value of  $m$  to a suitable degree of accuracy.  
 Give a reason for your answer.

$$\text{UB}(m) = \frac{\sqrt{\text{UB}(s)}}{\text{LB}(t)}$$

$$= \frac{\sqrt{3.475}}{8.1315}$$

$$= 0.\underline{2}\underline{2}\underline{9}\underline{2}\underline{4}\underline{8}\underline{6}\underline{2}\underline{4}$$

$$\text{LB}(m) = \frac{\sqrt{\text{LB}(s)}}{\text{UB}(t)}$$

$$= \frac{\sqrt{3.465}}{8.1325}$$

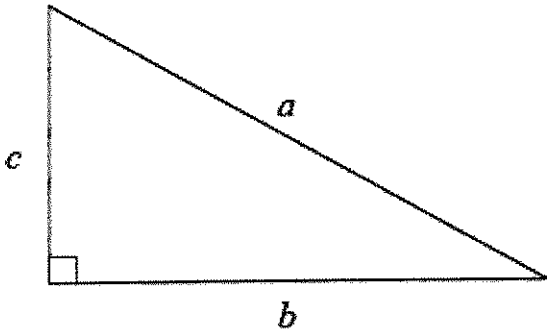
$$= 0.\underline{2}\underline{2}\underline{8}\underline{8}\underline{9}\underline{0}\underline{3}\underline{8}\underline{3}$$

$m = 0.229$  as this is correct for both  
 the upper bound and lower bound.

(Total for question = 5 marks)



8.  $a$  is 8.3 cm correct to the nearest mm  
 $b$  is 6.1 cm correct to the nearest mm



Calculate the upper bound for  $c$ .  
You must show your working.

$$c^2 = a^2 - b^2$$

$$\begin{aligned} \text{UB}(c) &= \sqrt{\text{UB}(a)^2 - \text{LB}(b)^2} \\ &= \sqrt{8.35^2 - 6.05^2} \\ &= \end{aligned}$$

..... 5.754997828 ..... cm  
(Total for question = 4 marks)

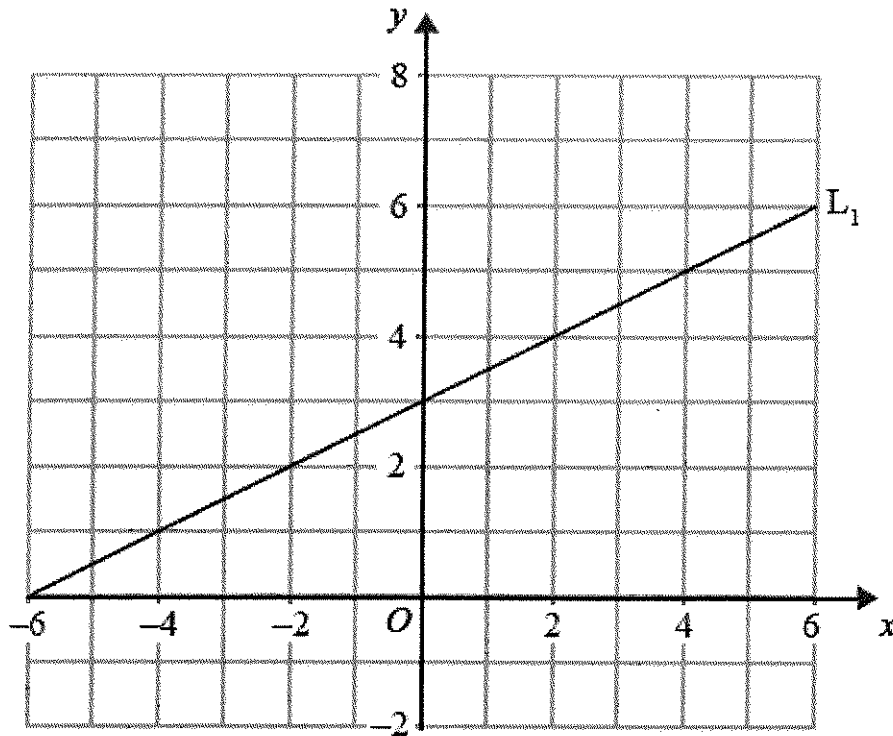
## Parallel and Perpendicular Graphs

### Things to remember:

- The general equation of a linear graph is given by  $y = mx + c$ , where  $m$  is the gradient and  $c$  is the  $y$ -intercept.
- Parallel graphs have the same gradient.
- Gradients of perpendicular graphs have a product of  $-1$ .

### Questions:

1. The diagram shows a straight line,  $L_1$ , drawn on a grid.



A straight line,  $L_2$ , is parallel to the straight line  $L_1$  and passes through the point  $(0, -5)$ . Find an equation of the straight line  $L_2$ .

$$y = \frac{1}{2}x - 5$$

(Total for Question is 3 marks)

2. The straight line L has equation  $y = 2x - 5$   
Find an equation of the straight line perpendicular to L which passes through  $(-2, 3)$ .

$$y = -\frac{1}{2}x + c$$

$$3 = -\frac{1}{2}(-2) + c$$

$$c = 2$$

$$y = -\frac{1}{2}x + 2$$

(Total for Question is 3 marks)

3. In the diagram, ABC is the line with equation  $y = -\frac{1}{2}x + 5$   
 $AB = BC$   
D is the point with coordinates  $(-13, 0)$

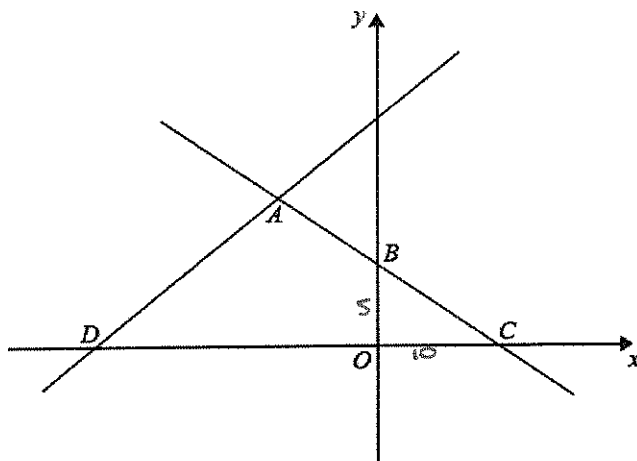


Diagram NOT  
accurately drawn

$$A(-10, 10)$$

Find an equation of the line through A and D.

$$m = \frac{10 - 0}{-10 - 13} = -\frac{5}{13}$$

$$y = mx + c$$

$$0 = -\frac{5}{13}(-13) + c$$

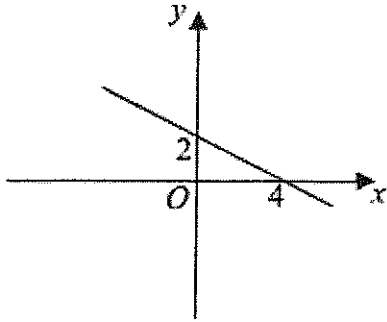
$$c = -5$$

$$y = -\frac{5}{13}x - 5$$

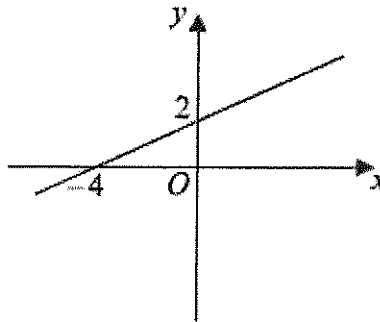
(Total for question = 5 marks)

4. Here are the graphs of 6 straight lines.

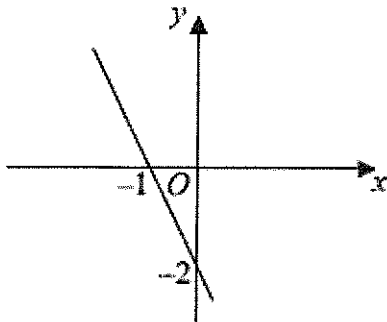
Graph A



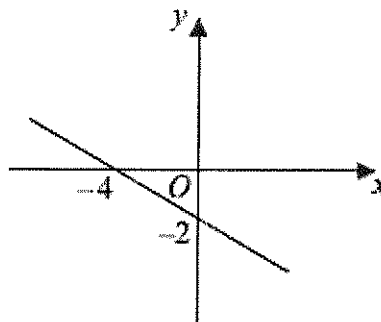
Graph B



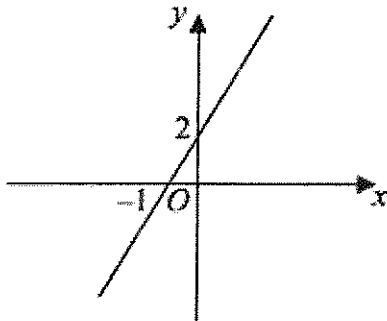
Graph C



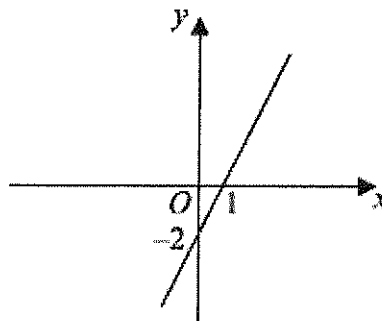
Graph D



Graph E



Graph F



Match each of the graphs A, B, C, D, E and F to the equations in the table.

<b>Equation</b>	$y = \frac{1}{2}x + 2$	$y = 2x - 2$	$y = -\frac{1}{2}x + 2$	$y = -2x - 2$	$y = 2x + 2$	$y = -\frac{1}{2}x - 2$
<b>Graph</b>	B	F	A	C	E	D

(Total for Question is 3 marks)

5. In the diagram, A is the point (-2, 0)  
 B is the point (0, 4)  
 C is the point (5, -1)

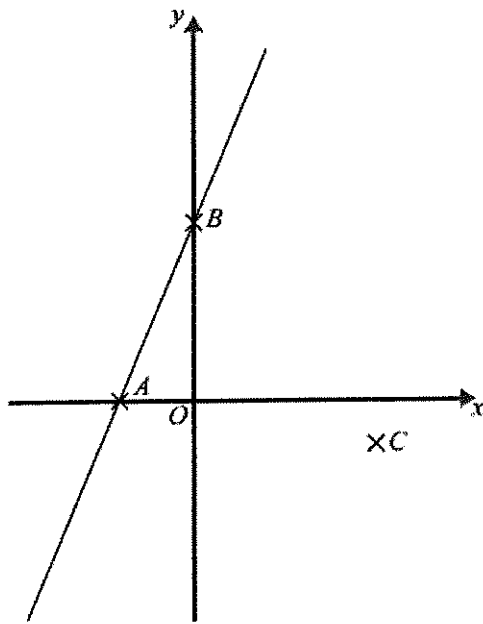


Diagram NOT  
 accurately drawn

Find an equation of the line that passes through C and is perpendicular to AB.

$$\text{Gradient of } AB = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$$

$$\text{Gradient of } L_2 = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + c$$

$$-1 = -\frac{1}{2}(5) + c$$

$$\frac{3}{2} = c$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

(Total for Question is 4 marks)

6. Find an equation of the straight line that is perpendicular to the straight line  $x + 2y = 5$  and that passes through the point (3, 7).

$$x + 2y = 5$$

$$2y = -x + 5$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

Perpendicular gradient = 2

$$y = 2x + c$$

$$7 = 2(3) + c$$

$$c = 1$$

$$y = 2x + 1$$

(Total for Question is 4 marks)

- \*7. **A** and **B** are straight lines.  
 Line **A** has equation  $2y = 3x + 8$   
 Line **B** goes through the points  $(-1, 2)$  and  $(2, 8)$

Do lines **A** and **B** intersect?  
 You must show all your working.

$$A \rightarrow y = \frac{3}{2}x + 4 \quad \text{Gradient} = \frac{3}{2}$$

$$B \rightarrow \text{gradient} = \frac{8 - 2}{2 - (-1)} = \frac{6}{3} = 2$$

Gradients are different  $\therefore$  lines are not parallel  
 $\therefore$  lines will intersect.

(Total for Question is 3 marks)

8. A straight line, **L**, is perpendicular to the line with equation  $y = 1 - 3x$ .  
 The point with coordinates  $(6, 3)$  is on the line **L**.  
 Find an equation of the line **L**.

$$y = -3x + 1$$

$$\text{Gradient of } L_1 = -3$$

$$\text{Gradient of } L_2 = \frac{1}{3}$$

$$y = \frac{1}{3}x + c$$

$$3 = \frac{1}{3}(6) + c$$

$$1 = c$$

$$\dots\dots\dots y = \frac{1}{3}x + 1 \dots\dots\dots$$

(Total for Question is 3 marks)

## Transformations of graphs

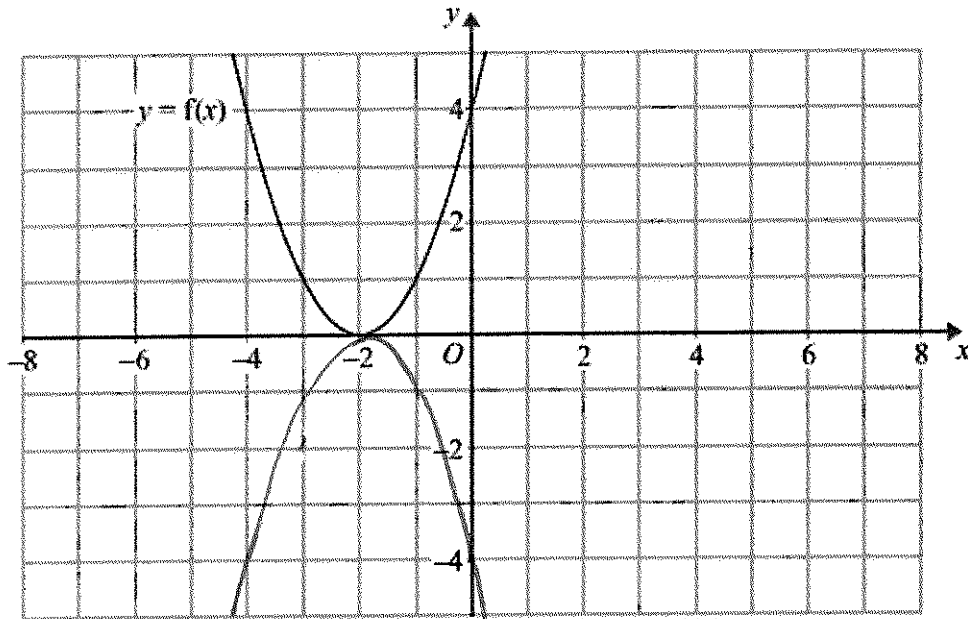
### Things to remember:

- $f(x)$  means the function of  $x$ .
- $-f(x)$  is a reflection in the  $x$ -axis.
- $f(-x)$  is a reflection in the  $y$ -axis.
- $f(x - a)$  is a translation in the  $x$ -axis,  $a$  units.
- $f(x) + b$  is a translation in the  $y$ -axis,  $b$  units.
- $cf(x)$  is an enlargement in the  $y$ -axis, scale factor  $c$ .
- $f(dx)$  is an enlargement in the  $x$ -axis, scale factor  $\frac{1}{d}$ .

### Questions:

1.  $y = f(x)$

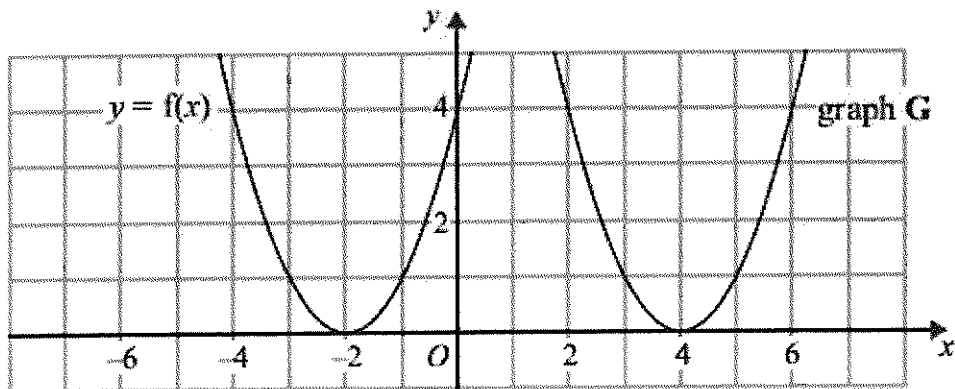
The graph of  $y = f(x)$  is shown on the grid.



(a) On the grid above, sketch the graph of  $y = -f(x)$ .

(2)

The graph of  $y = f(x)$  is shown on the grid.



The graph **G** is a translation of the graph of  $y = f(x)$ .

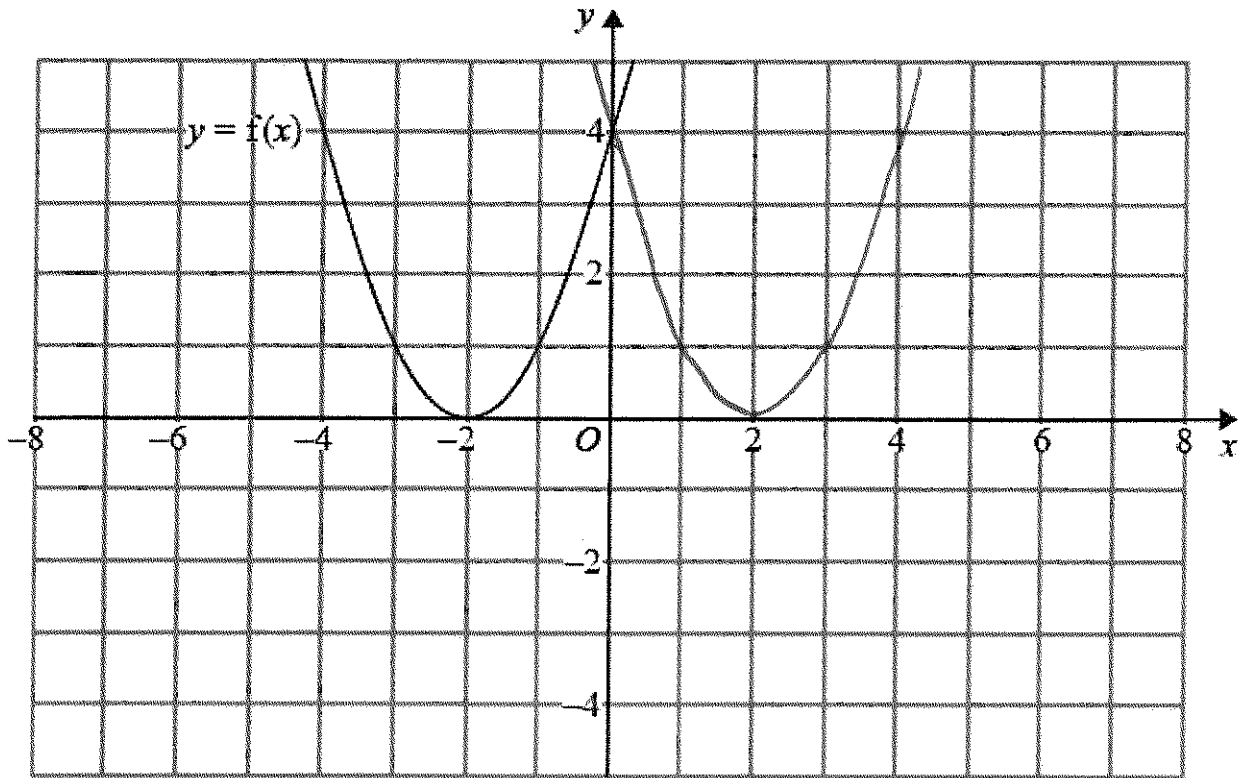
(b) Write down the equation of graph **G**.

.....  $f(x-6)$  .....

(2)

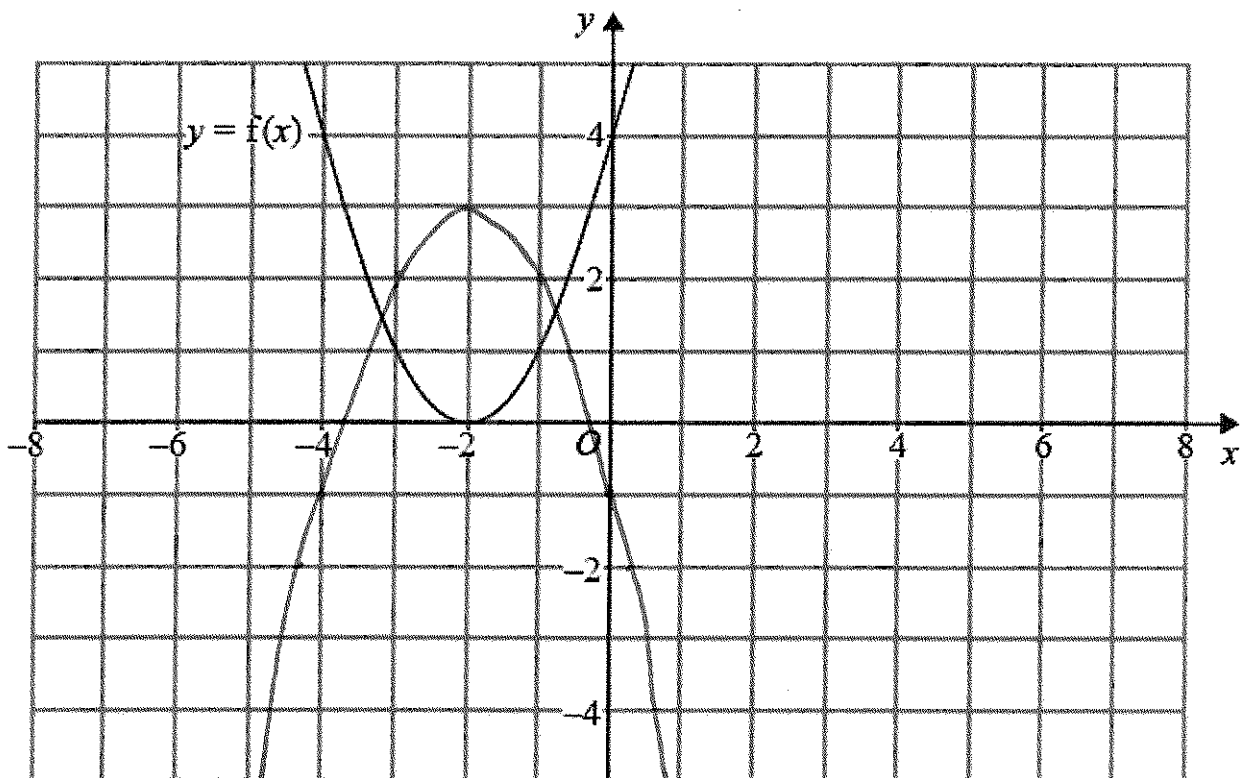
(Total for Question is 3 marks)

2. The graph of  $y = f(x)$  is shown on both grids below.



- (a) On the grid above, sketch the graph of  $y = f(-x)$

(1)



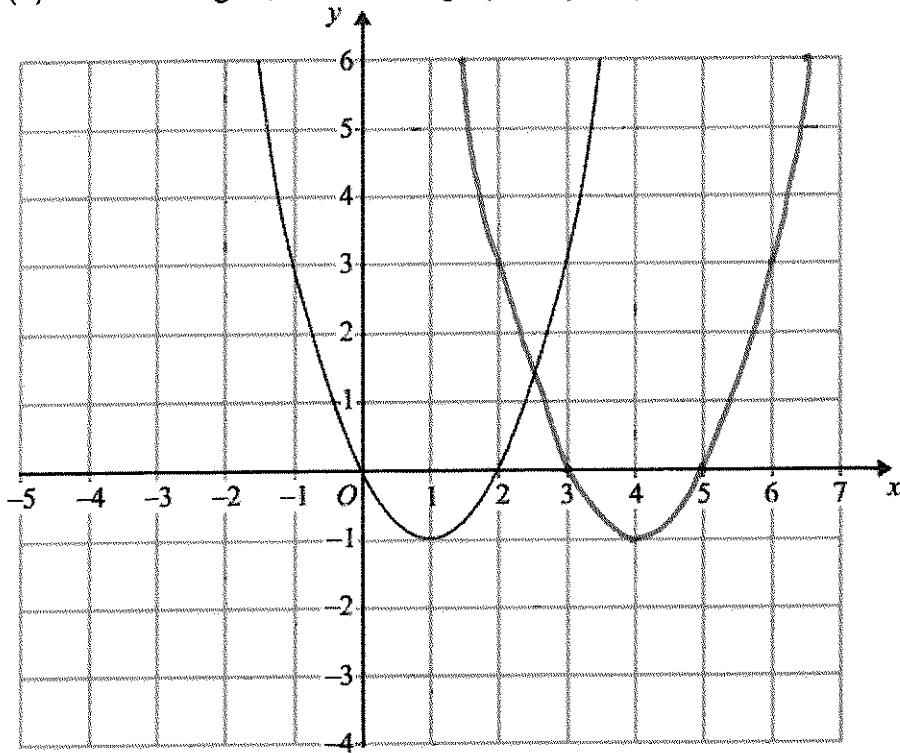
- (b) On this grid, sketch the graph of  $y = -f(x) + 3$

(1)

(Total for question = 2 marks)

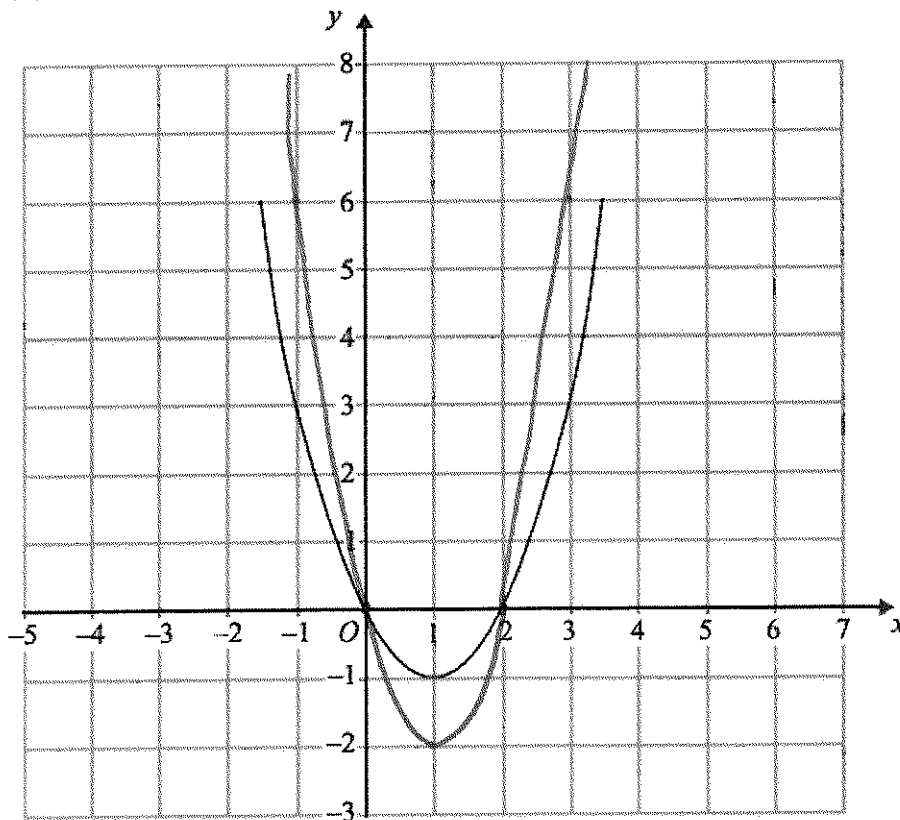


3. The graph of  $y = f(x)$  is shown on each of the grids.  
 (a) On this grid, sketch the graph of  $y = f(x - 3)$



(2)

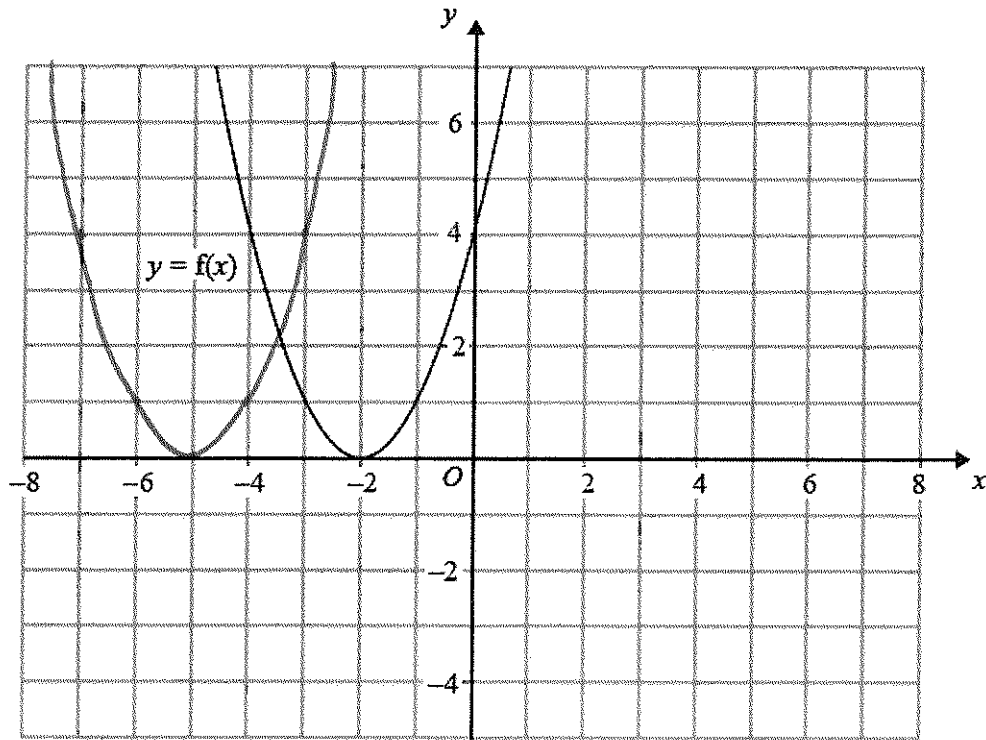
- (b) On this grid, sketch the graph of  $y = 2f(x)$



(2)

(Total for Question is 4 marks)

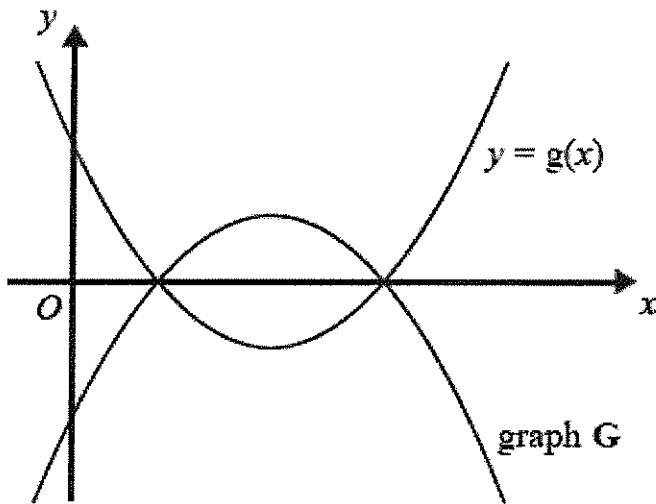
4. The graph of  $y = f(x)$  is shown on the grid.



(a) On the grid above, sketch the graph of  $y = f(x + 3)$

(2)

The graph of  $y = g(x)$  is shown below.



The graph **G** is the reflection of  $y = g(x)$  in the  $x$ -axis.

(b) Write down an equation of graph **G**.

$y = -f(x)$

(1)

(Total for question = 3 marks)

## Algebraic Fractions – Simplifying

### Things to remember:

- Factorise the numerator and denominator;
- Cancel common factors;
- Then add/subtract/multiply divide if necessary.

### Questions:

1. Simplify  $\frac{p^2-9}{2p+6}$

$$\frac{(p+3)(p-3)}{2(p+3)}$$

$$\frac{p-3}{2}$$

.....  
(Total 3 marks)

2. Simplify fully  $\frac{6x^2+3x}{4x^2-1}$

$$\frac{3x(2x+1)}{(2x+1)(2x-1)}$$

$$\frac{3x}{2x-1}$$

.....  
(Total 3 marks)

3. Simplify  $\frac{x^2+2x+1}{x^2+3x+2}$

$$\frac{(x+1)(x+1)}{(x+1)(x+2)}$$

$$\frac{x+1}{x+2}$$

.....  
(Total 3 marks)

4. Simplify fully  $\frac{x^2+x-6}{x^2-7x+10}$

$$\frac{(x+3)(\cancel{x-2})}{(\cancel{x-2})(x-5)}$$

$$\frac{x+3}{x-5}$$

(Total 3 marks)

5. Simplify fully  $\frac{x^2-8x+15}{2x^2-7x-15}$

$$\frac{(x-5)(\cancel{x-3})}{(\cancel{x-5})(2x+3)}$$

	$2x$	$3$
$x$	$2x^2$	$3x$
$-5$	$-10x$	$-15$

$$\frac{x-3}{2x+3}$$

(Total 3 marks)

6. Simplify fully  $\frac{2x^2+3x+1}{x^2-3x-4}$

$$\frac{(2x+1)(\cancel{x+1})}{(x-4)(\cancel{x+1})}$$

	$2x$	$1$
$2x$	$2x^2$	$2x$
$1$	$x$	$+1$

$$\frac{2x+1}{x-4}$$

(Total 3 marks)

7. (a) Simplify  $\frac{2x+4}{x^2+4x+4}$

$$\frac{2(x+2)}{(x+2)(x+2)}$$

$$\frac{2}{x+2}$$

(3)

(b) Write  $\frac{1}{x+4} + \frac{2}{x-4}$  as a single fraction in its simplest form.

$$\frac{1(x-4) + 2(x+4)}{x^2-16}$$

$$\frac{x-4 + 2x+8}{x^2-16}$$

$$\frac{3x+4}{x^2-16}$$

(3)

(Total 6 marks)

8. Simplify fully  $\frac{x+3}{4} + \frac{x-5}{3}$

$$\frac{3(x+3) + 4(x-5)}{12}$$

$$\frac{3x+9+4x-20}{12}$$

$$\frac{7x-11}{12}$$

(Total 3 marks)

## Algebraic fractions – solving

### Things to remember:

- Multiply every term by the product of the denominators;
- Solve to find  $x$ .

### Questions:

1. Solve  $\frac{5(2x+1)}{3} = 4x + 7$

$$5(2x+1) = 3(4x+7)$$

$$10x + 5 = 12x + 21$$

$$-16 = 2x$$

$$-8 = x$$

$x = \dots\dots\dots -8 \dots\dots\dots$   
(Total 3 marks)

2. (a) Show that the equation  $\frac{5}{x+2} = \frac{4-3x}{x-1}$

can be rearranged to give  $3x^2 + 7x - 13 = 0$

$$5(x-1) = (4-3x)(x+2)$$

$$5x - 5 = 4x + 8 - 3x^2 - 6x$$

$$3x^2 + 7x - 13 = 0$$

$$\frac{-7 \pm \sqrt{7^2 - 4(3)(-13)}}{6}$$

(b) Solve  $3x^2 + 7x - 13 = 0$   
Give your solutions correct to 2 decimal places.

$x = \dots\dots\dots 1.39 \dots\dots\dots$  or  $x = \dots\dots\dots -3.72 \dots\dots\dots$

(3)  
(Total 6 marks)

3. Solve the equation  $\frac{x}{2x-3} + \frac{4}{x+1} = 1$

$$x(x+1) + 4(2x-3) = (2x-3)(x+1)$$

$$\cancel{x^2} + \cancel{x} + 8x - 12 = \cancel{2x^2} - 3x + \cancel{2x} - 3$$

$$0 = x^2 - 10x + 9$$

$$0 = (x-9)(x-1)$$

$x = \dots 9 \text{ or } 1 \dots$   
(Total 5 marks)

4. Solve the equation  $\frac{3}{x+3} - \frac{4}{x-3} = \frac{5x}{x^2-9}$

$$3(x-3) - 4(x+3) = \frac{5x(x^2-9)}{\cancel{x^2-9}}$$

$$3x - 9 - 4x - 12 = 5x$$

$$-21 = 6x$$

$$-\frac{7}{2} = x$$

$x = \dots -\frac{7}{2} \text{ (or } -3.5) \dots$   
(Total 4 marks)

5. (a) Solve  $\frac{3}{x} + \frac{3}{2x} = 2$

$$3(2x) + 3(x) = 2(x)(2x)$$

$$6x + 3x = 4x^2$$

$$0 = 4x^2 - 9x$$

$$0 = x(4x - 9)$$

$$x = \dots 0 \dots \text{or} \dots \frac{9}{4} \dots$$

(2)

(b) Using your answer to part (a), or otherwise, solve  $\frac{3}{(y-1)^2} + \frac{3}{2(y-1)^2} = 2$

$$(y-1)^2 = \frac{9}{4}$$

$$y-1 = \frac{3}{2} \quad \text{or} \quad -\frac{3}{2}$$

$$y = \frac{5}{2} \quad \text{or} \quad -\frac{1}{2}$$

$$y = \dots \frac{5}{2} \dots \text{or} \dots -\frac{1}{2} \dots$$

(3)

(Total 5 marks)



## Solving Quadratic Inequalities

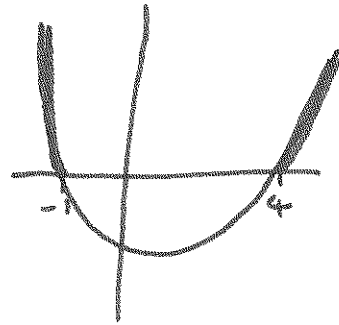
### Things to remember:

- Start by solving the quadratic to find the values of  $x$ , then sketch the graph to determine the inequality.

### Questions:

1. Solve  $x^2 > 3x + 4$

$$x^2 - 3x - 4 > 0$$
$$(x - 4)(x + 1) > 0$$

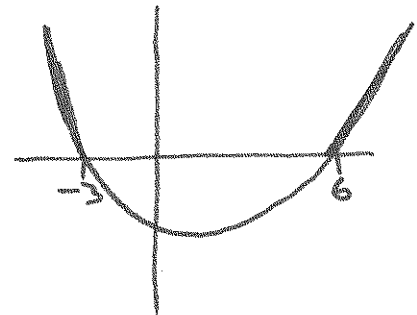


$$x > 4 \text{ or } x < -1$$

(Total for question = 3 marks)

2. Solve the inequality  $x^2 > 3(x + 6)$

$$x^2 > 3x + 18$$
$$x^2 - 3x - 18 > 0$$
$$(x - 6)(x + 3) > 0$$

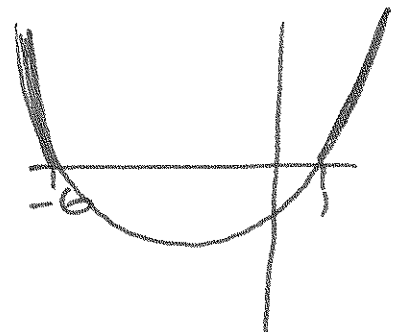


$$x < -3, x > 6$$

(Total for question = 4 marks)

3. Solve the inequality  $x^2 + 5x > 6$

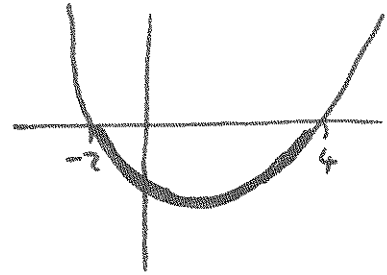
$$x^2 + 5x - 6 > 0$$
$$(x + 6)(x - 1) > 0$$



$$x < -6, x > 1$$

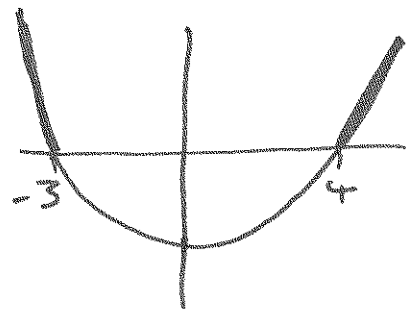
(Total for question = 3 marks)

4. Solve the inequality  $x^2 - 2x + 8 < 0$   
 $(x-4)(x+2) < 0$



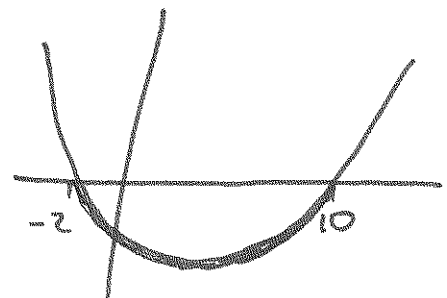
$-2 < x < 4$   
 .....  
 (Total for question = 3 marks)

5. Solve the inequality  $x^2 - x \geq 12$   
 $x^2 - x - 12 \geq 0$   
 $(x-4)(x+3) \geq 0$



$x < -3, x > 4$   
 .....  
 (Total for question = 3 marks)

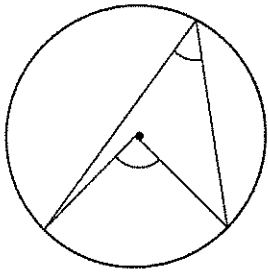
6. Solve the inequality  $x^2 \leq 4(2x + 5)$   
 $x^2 \leq 8x + 20$   
 $x^2 - 8x - 20 \leq 0$   
 $(x-10)(x+2) \leq 0$



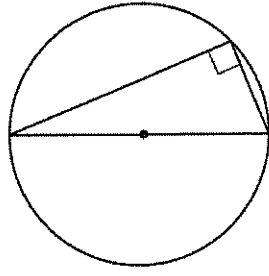
$-2 \leq x \leq 10$   
 .....  
 (Total for question = 4 marks)

# Circle theorems

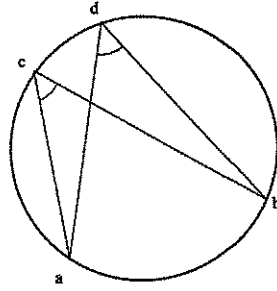
## Things to remember:



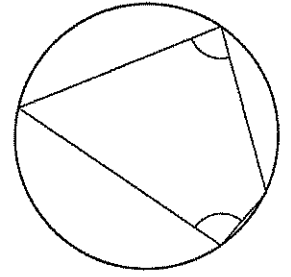
The angle at the centre is twice the angle at the circumference.



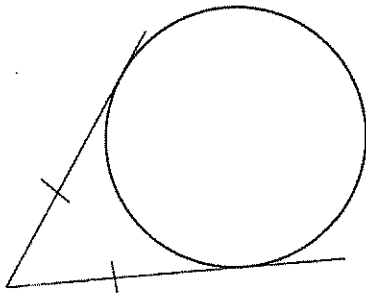
The angle in a semi-circle is  $90^\circ$ .



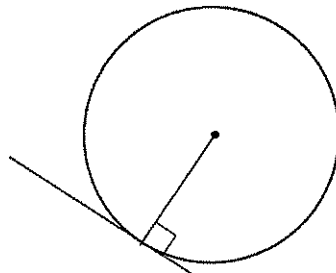
Angles subtended by the same arc are equal.



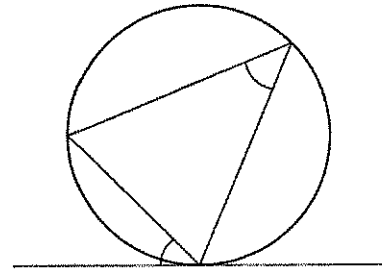
Opposite angles in a cyclic quadrilateral sum to  $180^\circ$ .



Tangents from a point are equal.



A tangent is perpendicular to a radius.



Angles in alternate segments are equal.

## Questions:

1.

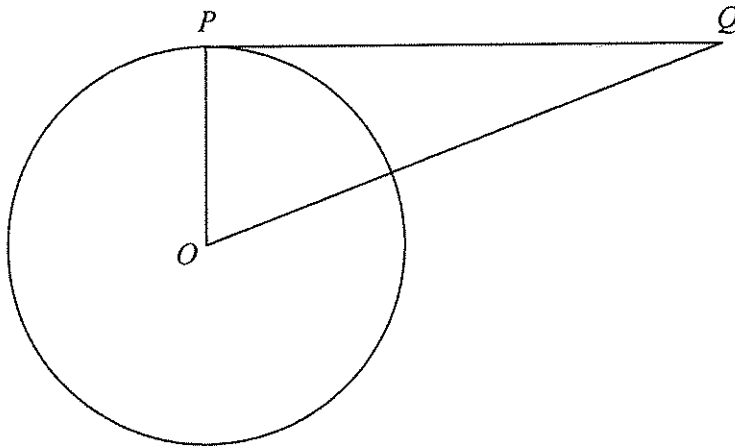


Diagram **NOT** accurately drawn

$P$  is a point on the circumference of the circle, centre  $O$ .

$PQ$  is a tangent to the circle.

(i) Write down the size of angle  $OPQ$ .

90

(ii) Give a reason for your answer.

Tangent and radius meet at a right-angle.

(Total 2 marks)

2.

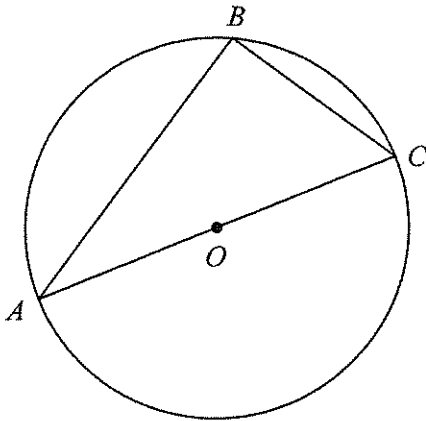


Diagram **NOT** accurately drawn

A, B and C are points on the circumference of a circle, centre O.

AC is a diameter of the circle.

(a) (i) Write down the size of angle ABC.

..... 90 .....

(ii) Give a reason for your answer.

..... Tangents and radii meet at a right-angle .....

(2)

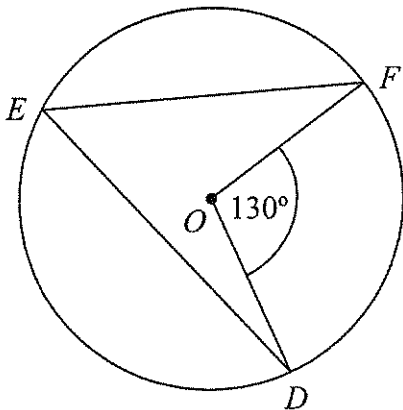


Diagram **NOT** accurately drawn

D, E and F are points on the circumference of a circle, centre O.

Angle  $DOF = 130^\circ$ .

(b) (i) Work out the size of angle DEF.

..... 65 .....

(ii) Give a reason for your answer.

..... Angles at the circumference are half at the centre when subtended by same arc .....

(2)

(Total 4 marks)

3.

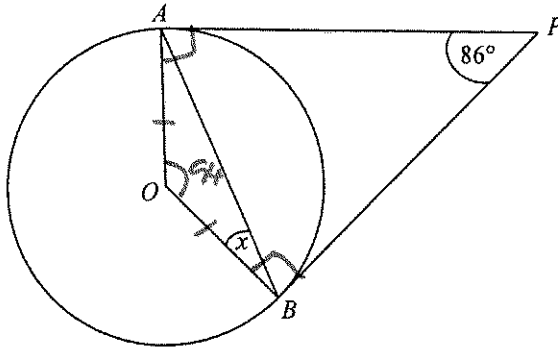


Diagram **NOT** accurately drawn  
 A and B are points on the circumference of a circle, centre O.  
 PA and PB are tangents to the circle.  
 Angle APB is  $86^\circ$ .  
 Work out the size of the angle marked x.

..... 43 .....  
 (Total 2 marks)

4.

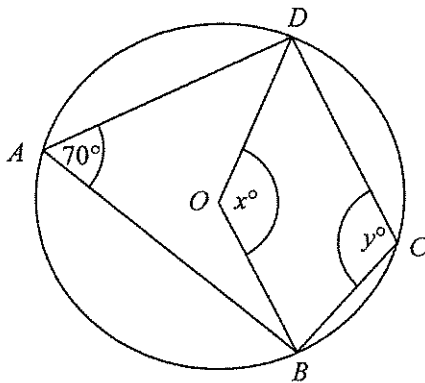


Diagram **NOT** accurately drawn  
 In the diagram, A, B, C and D are points on the circumference of a circle, centre O.  
 Angle  $BAD = 70^\circ$ .  
 Angle  $BOD = x^\circ$ .  
 Angle  $BCD = y^\circ$ .

(a) (i) Work out the value of x.

..... 140 .....  
 (2)

(ii) Give a reason for your answer.

..... Angles at centre are double that at  
 ..... circumference when subtended by same arc.  
 (2)

(b) (i) Work out the value of y.

..... 110 .....  
 (2)

(ii) Give a reason for your answer.

..... Opposite angles in a cyclic quadrilateral  
 ..... sum to  $180^\circ$ .  
 (2)

(Total 4 marks)

5.

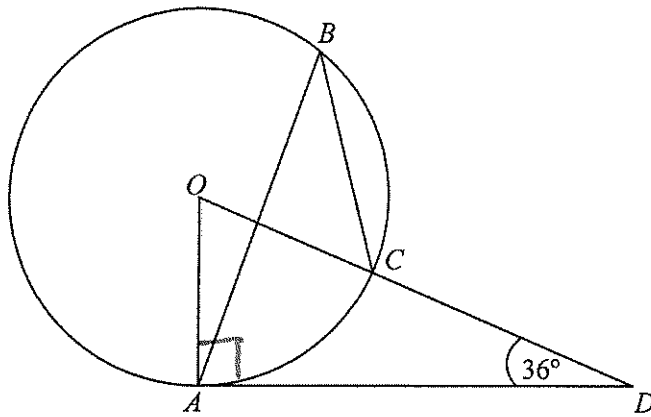


Diagram **NOT** accurately drawn  
The diagram shows a circle centre  $O$ .  
 $A$ ,  $B$  and  $C$  are points on the circumference.  
 $DCO$  is a straight line.  
 $DA$  is a tangent to the circle.  
Angle  $ADO = 36^\circ$

(a) Work out the size of angle  $AOD$ .

54

(2)

(b) (i) Work out the size of angle  $ABC$ .

27

(ii) Give a reason for your answer.

Angles at the circumference are half that  
at the centre when subtended by same arc.

(3)

(Total 5 marks)

## Vectors

### Things to remember:

- Use the letter provided in the question.
- Going against the arrow is a negative.
- Vectors need to be written in bold or underlined.
- They can be manipulated similarly to algebra.

### Questions:

1. The diagram shows a regular hexagon  $ABCDEF$  with centre  $O$ .

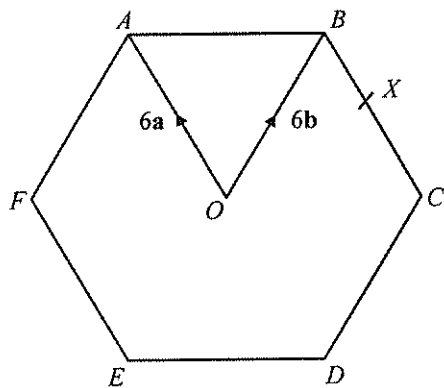


Diagram NOT accurately drawn

$$\overrightarrow{OA} = 6\mathbf{a} \quad \overrightarrow{OB} = 6\mathbf{b}$$

- (a) Express in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$

(i)  $\overrightarrow{AB}$ ,

$$\dots\dots\dots -6\mathbf{a} + 6\mathbf{b}$$

(ii)  $\overrightarrow{EF}$ .

$$\dots\dots\dots 6\mathbf{a}$$

(2)

$X$  is the midpoint of  $BC$ .

- (b) Express  $\overrightarrow{EX}$  in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$

$$\begin{aligned} \overrightarrow{EX} &= \overrightarrow{EB} + \frac{1}{2} \overrightarrow{BC} \\ &= 12\mathbf{b} + \frac{1}{2}(-6\mathbf{a}) \\ &= 12\mathbf{b} - 3\mathbf{a} \end{aligned}$$

$$\dots\dots\dots 12\mathbf{b} - 3\mathbf{a}$$

(2)

$Y$  is the point on  $AB$  extended, such that  $AB : BY = 3 : 2$

- (c) Prove that  $E$ ,  $X$  and  $Y$  lie on the same straight line.

$$\begin{aligned} \overrightarrow{XY} &= \overrightarrow{XB} + \frac{2}{3} \overrightarrow{AB} \\ &= 3\mathbf{a} + \frac{2}{3}(-6\mathbf{a} + 6\mathbf{b}) \\ &= 3\mathbf{a} - 4\mathbf{a} + 4\mathbf{b} \\ &= 4\mathbf{b} - \mathbf{a} \end{aligned}$$

$$\overrightarrow{EX} = 3(4\mathbf{b} - \mathbf{a})$$

Since  $\overrightarrow{XY}$  and  $\overrightarrow{EX}$  share a factor, they are parallel. Since they share point  $X$ , they are therefore on a straight line.

(3)

(Total 7 marks)

2.  $T$  is the point on  $PQ$  for which  $PT : TQ = 2 : 1$ .

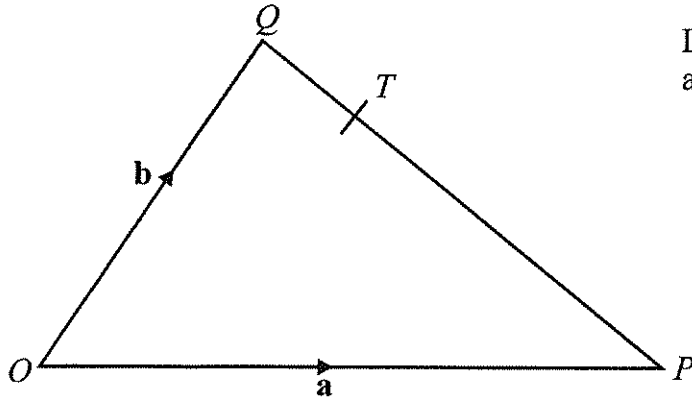


Diagram **NOT** accurately drawn

$OPQ$  is a triangle.

$$\overrightarrow{OP} = \mathbf{a} \text{ and } \overrightarrow{OQ} = \mathbf{b}.$$

- (a) Write down, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , an expression for  $\overrightarrow{PQ}$ .

$$\overrightarrow{PQ} = \dots -\mathbf{a} + \mathbf{b} \dots \quad (1)$$

- (b) Express  $\overrightarrow{OT}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .  
Give your answer in its simplest form.

$$\begin{aligned} \overrightarrow{OT} &= \overrightarrow{OQ} + \frac{1}{3} \overrightarrow{QP} \\ &= \mathbf{b} + \frac{1}{3}(\mathbf{a} - \mathbf{b}) \\ &= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} \end{aligned}$$

$$\overrightarrow{OT} = \dots \frac{1}{3}(\mathbf{a} + 2\mathbf{b}) \dots \quad (2)$$

(Total 3 marks)



3. OABC is a parallelogram.

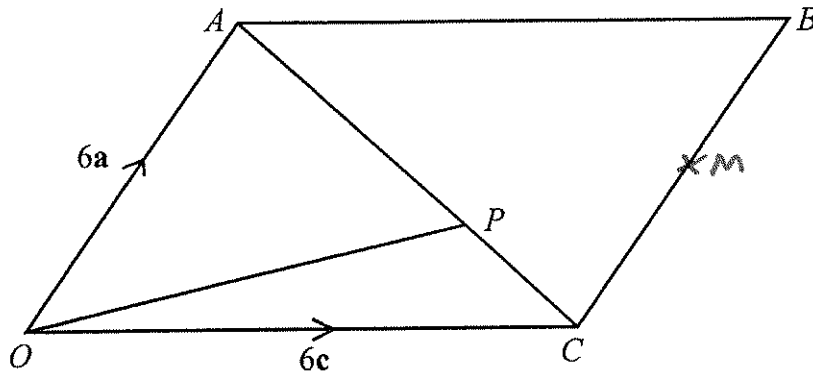


Diagram NOT  
accurately drawn

$P$  is the point on  $AC$  such that  $AP = \frac{2}{3} AC$ .

$$\vec{OA} = 6\mathbf{a}, \quad \vec{OC} = 6\mathbf{c}.$$

- (a) Find the vector  $\vec{OP}$ .  
Give your answer in terms of  $\mathbf{a}$  and  $\mathbf{c}$ .

$$\begin{aligned} \vec{OP} &= \vec{OC} + \vec{CP} \\ &= 6\mathbf{c} + \frac{1}{3}(6\mathbf{c} - 6\mathbf{a}) \\ &= 6\mathbf{c} + 2\mathbf{a} - 2\mathbf{c} \end{aligned}$$

$$4\mathbf{c} + 2\mathbf{a}$$

(3)

The midpoint of  $CB$  is  $M$ .

- (b) Prove that  $OPM$  is a straight line.

$$\begin{aligned} \vec{OM} &= \vec{OC} + \frac{1}{2} \vec{OB} \\ &= 6\mathbf{c} + 3\mathbf{a} \\ &= 3(2\mathbf{c} + \mathbf{a}) \end{aligned}$$

$$\vec{OP} = 2(2\mathbf{c} + \mathbf{a})$$

$\vec{OM}$  and  $\vec{OP}$  share a factor so are parallel.  
They share point  $O$   $\therefore$  are a straight line.

(2)

(Total 5 marks)

4.  $OPQ$  is a triangle.  
 $R$  is the midpoint of  $OP$ .  
 $S$  is the midpoint of  $PQ$ .  
 $\vec{OP} = \mathbf{p}$  and  $\vec{OQ} = \mathbf{q}$

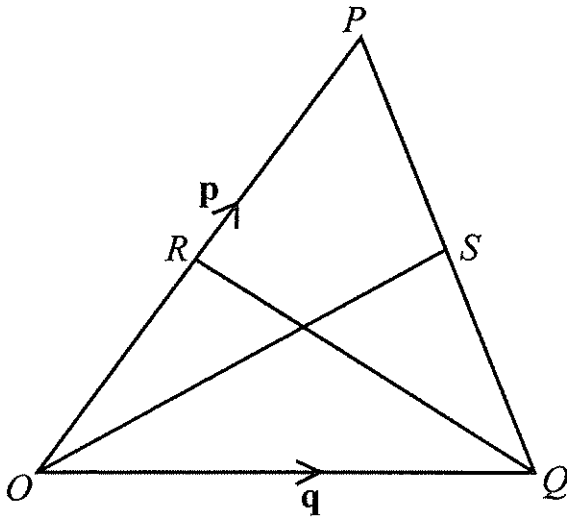


Diagram NOT  
accurately drawn

- (i) Find  $\vec{OS}$  in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

$$\begin{aligned}\vec{OS} &= \vec{OQ} + \frac{1}{2} \vec{QP} \\ &= \mathbf{q} + \frac{1}{2} (\mathbf{p} - \mathbf{q}) \\ &= \frac{1}{2} (\mathbf{p} + \mathbf{q})\end{aligned}$$

$$\vec{OS} = \frac{1}{2} (\mathbf{p} + \mathbf{q})$$

- (ii) Show that  $RS$  is parallel to  $OQ$ .

$$\begin{aligned}\vec{OQ} &= \mathbf{q} \\ \vec{RS} &= \vec{RO} + \vec{OQ} + \vec{QS} \\ &= -\frac{1}{2} \mathbf{p} + \mathbf{q} + \frac{1}{2} (\mathbf{p} - \mathbf{q}) \\ &= \frac{1}{2} \mathbf{q}\end{aligned}$$

$\mathbf{q}$  is a factor of both vectors  $\therefore \vec{OQ}$   
and  $\vec{RS}$  are parallel.

(Total 5 marks)

5.  $OPQR$  is a trapezium with  $PQ$  parallel to  $OR$ .

$$\overrightarrow{OP} = 2\mathbf{b} \quad \overrightarrow{PQ} = 2\mathbf{a} \quad \overrightarrow{OR} = 6\mathbf{a}$$

$M$  is the midpoint of  $PQ$  and  $N$  is the midpoint of  $OR$ .

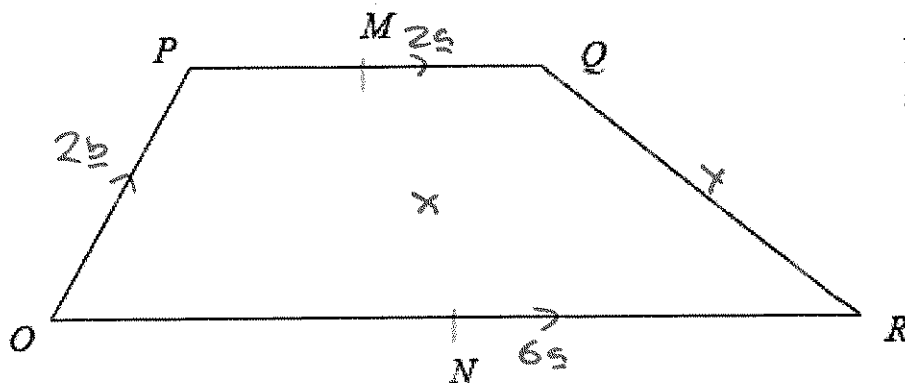


Diagram NOT  
accurately drawn

- (a) Find the vector  $\overrightarrow{MN}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\begin{aligned} \overrightarrow{MN} &= \frac{1}{2} \overrightarrow{QP} + \overrightarrow{PO} + \frac{1}{2} \overrightarrow{OR} \\ &= -\mathbf{a} - 2\mathbf{b} + 3\mathbf{a} \end{aligned}$$

$$\overrightarrow{MN} = 2\mathbf{a} - 2\mathbf{b} \quad \dots \dots \dots (2)$$

$X$  is the midpoint of  $MN$  and  $Y$  is the midpoint of  $QR$ .

- (b) Prove that  $XY$  is parallel to  $OR$ .

$$\begin{aligned} \overrightarrow{OR} &= 6\mathbf{a} \\ \overrightarrow{XY} &= \frac{1}{2} \overrightarrow{MN} + \frac{1}{2} \overrightarrow{OR} + \frac{1}{2} \overrightarrow{RQ} \\ &= \frac{1}{2} (2\mathbf{a} - 2\mathbf{b}) + 3\mathbf{a} + \frac{1}{2} (-6\mathbf{a} + 2\mathbf{b} + 2\mathbf{a}) \\ &= \mathbf{a} - \mathbf{b} + 3\mathbf{a} - 3\mathbf{a} + \mathbf{b} + \mathbf{a} \\ &= 2\mathbf{a} \end{aligned}$$

$\mathbf{a}$  is a factor of both vectors so they are parallel.

(2)  
(Total 4 marks)

6.  $ABCD$  is a straight line.

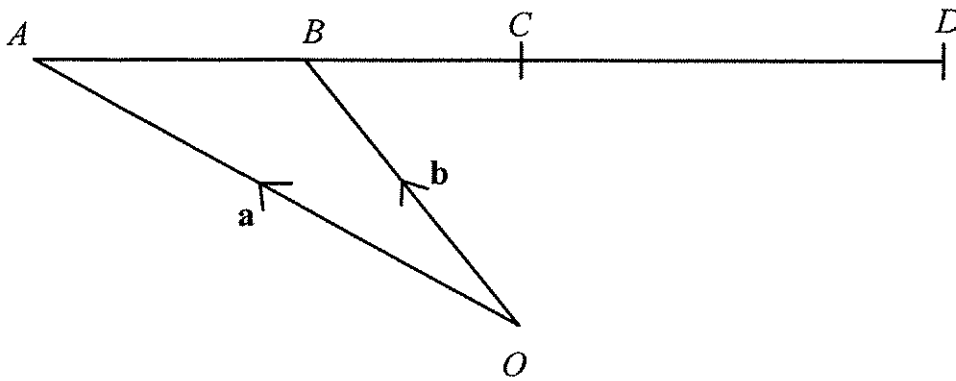


Diagram **NOT** accurately drawn

$O$  is a point so that  $\vec{OA} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$ .

$B$  is the midpoint of  $AC$ .

$C$  is the midpoint of  $AD$ .

Express, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the vectors

(i)  $\vec{AC}$

$$\vec{AC} = -\mathbf{a} + \mathbf{b} - \mathbf{a} + \mathbf{b}$$

$$\dots\dots\dots -2\mathbf{a} + 2\mathbf{b}$$

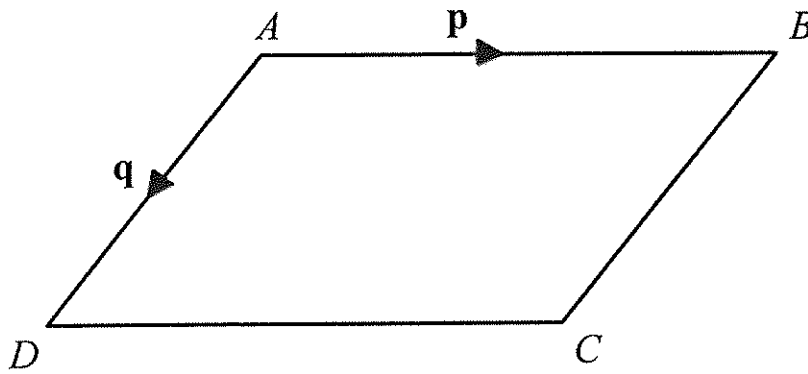
(ii)  $\vec{OD}$

$$\begin{aligned} \vec{OD} &= \vec{OA} + 2\vec{AC} \\ &= \mathbf{a} + 2(-2\mathbf{a} + 2\mathbf{b}) \\ &= \mathbf{a} - 4\mathbf{a} + 4\mathbf{b} \end{aligned}$$

$$\dots\dots\dots -3\mathbf{a} + 4\mathbf{b}$$

(Total 3 marks)

7. Diagram NOT accurately drawn



$ABCD$  is a parallelogram.

$AB$  is parallel to  $DC$ .

$AD$  is parallel to  $BC$ .

$$\vec{AB} = \mathbf{p}$$

$$\vec{AD} = \mathbf{q}$$

(a) Express, in terms of  $\mathbf{p}$  and  $\mathbf{q}$

(i)  $\vec{AC}$

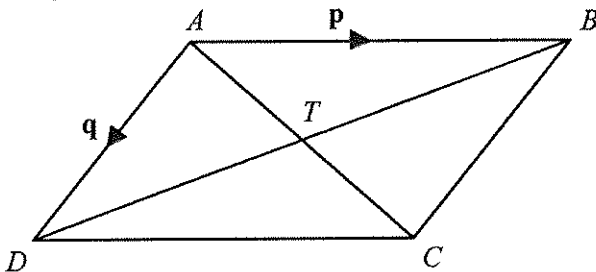
.....  $\mathbf{q} + \mathbf{p}$  .....

(ii)  $\vec{BD}$

.....  $-\mathbf{p} + \mathbf{q}$  .....

(2)

Diagram NOT accurately drawn



$AC$  and  $BD$  are diagonals of parallelogram  $ABCD$ .

$AC$  and  $BD$  intersect at  $T$ .

(b) Express  $\vec{AT}$  in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

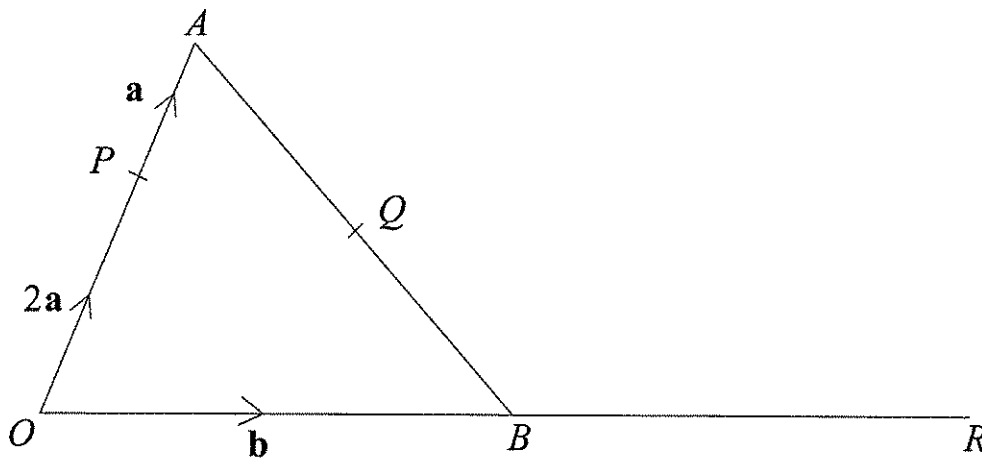
$$\vec{AT} = \frac{1}{2} \vec{AC}$$

.....  $\frac{1}{2}(\mathbf{q} + \mathbf{p})$  .....

(1)

(Total 3 marks)

8. Diagram **NOT** accurately drawn  
 OAB is a triangle.  
 B is the midpoint of OR.  
 Q is the midpoint of AB.  
 $\overrightarrow{OP} = 2\mathbf{a}$   $\overrightarrow{PA} = \mathbf{a}$   $\overrightarrow{OB} = \mathbf{b}$



- (a) Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the vectors

(i)  $\overrightarrow{AB}$ ,

.....  $-3\mathbf{a} + \mathbf{b}$  .....

(ii)  $\overrightarrow{PR}$ ,

.....  $-2\mathbf{a} + 2\mathbf{b}$  .....

(iii)  $\overrightarrow{PQ}$ .

$\mathbf{a} + \frac{1}{2}(-3\mathbf{a} + \mathbf{b})$   
 $= -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

.....  $\frac{1}{2}(\mathbf{b} - \mathbf{a})$  .....

(4)

- (b) Hence explain why PQR is a straight line.

$\mathbf{b} - \mathbf{a}$  is a factor of both so they are parallel. Both vectors share point P  $\therefore$  lie on the same straight line.

(2)

The length of PQ is 3 cm.

- (c) Find the length of PR.

SF 4

$3 \times 4 = 12$

..... 12 .....

cm

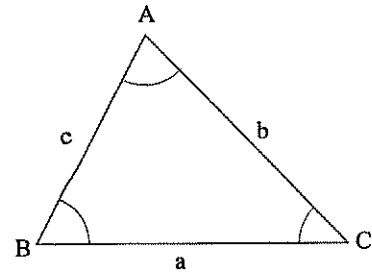
(1)

(Total 7 marks)

## Sine and Cosine Rules

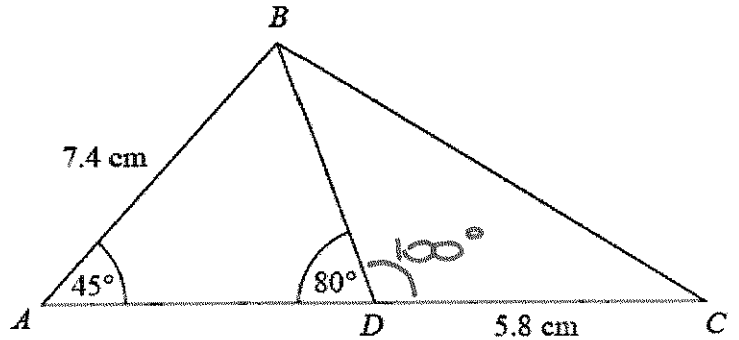
### Things to remember:

- For any triangle ABC,  $a^2 = b^2 + c^2 - 2bc \cos A$
- For any triangle ABC,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- For any triangle ABC, area =  $\frac{1}{2} a b \sin C$



### Questions:

1. Diagram **NOT** accurately drawn  
 ABC is a triangle.  
 D is a point on AC.  
 Angle BAD =  $45^\circ$   
 Angle ADB =  $80^\circ$   
 AB = 7.4 cm  
 DC = 5.8 cm  
 Work out the length of BC.  
 Give your answer correct to 3 significant figures.



$$\frac{BD}{\sin 45} = \frac{7.4}{\sin 80}$$

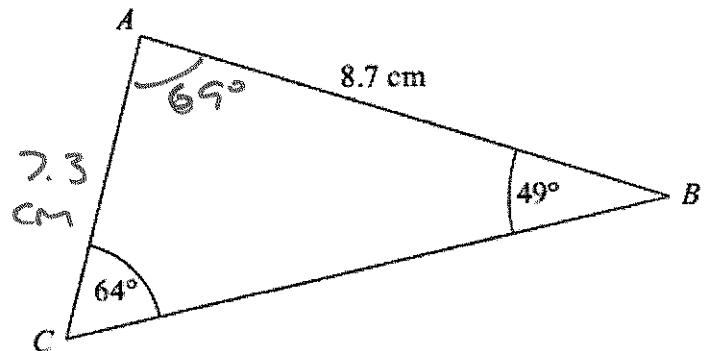
$$BD = 5.313311... \text{ cm}$$

$$BC^2 = 5.3^2 + 5.8^2 - (2 \times 5.3 \times 5.8 \times \cos 100)$$

$$BC = 8.51903... \text{ cm}$$

..... 8.52 ..... cm  
 (Total for question = 5 marks)

2. Diagram **NOT** accurately drawn  
 ABC is a triangle.  
 AB = 8.7 cm.  
 Angle ABC =  $49^\circ$ .  
 Angle ACB =  $64^\circ$ .  
 Calculate the area of triangle ABC.  
 Give your answer correct to 3 significant figures.



$$\frac{AC}{\sin 49} = \frac{8.7}{\sin 64}$$

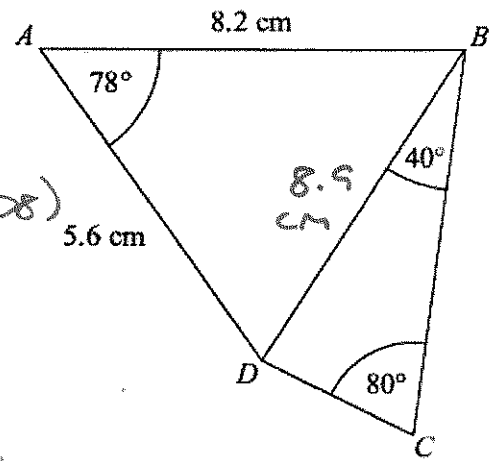
$$AC = 7.30531... \text{ cm}$$

$$\frac{1}{2} \times 7.305... \times 8.7 \times \sin 69$$

$$= 29.667429... \text{ cm}^2$$

..... 29.7 ..... cm<sup>2</sup>  
 (Total for Question is 5 marks)

3.  $ABCD$  is a quadrilateral.  
Diagram **NOT** accurately drawn  
Work out the length of  $DC$ .  
Give your answer correct to 3 significant figures.



$$BD^2 = 5.6^2 + 8.2^2 - (2 \times 5.6 \times 8.2 \times \cos 78)$$

$$BD = 8.9165... \text{ cm}$$

$$\frac{DC}{\sin 40} = \frac{8.9...}{\sin 80}$$

$$DC = 5.81988... \text{ cm}$$

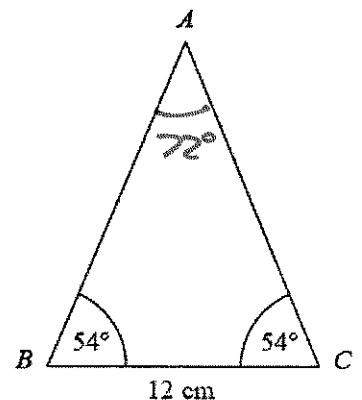
..... 5.82 ..... cm  
(Total for Question is 6 marks)

4. Diagram **NOT** accurately drawn  
 $ABC$  is an isosceles triangle.  
Work out the area of the triangle.  
Give your answer correct to 3 significant figures.

$$\frac{AB}{\sin 54} = \frac{12}{\sin 72}$$

$$AB = 10.2078... \text{ cm}$$

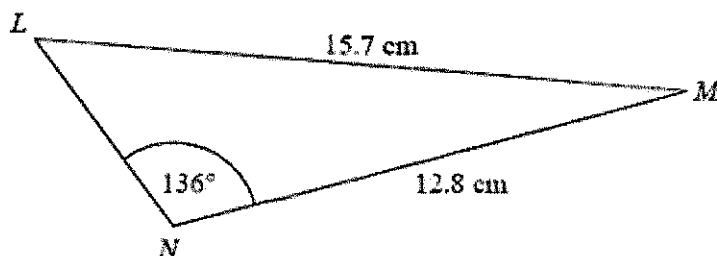
$$\begin{aligned} & \frac{1}{2} \times 10.2...^2 \times \sin 72 \\ & = 49.5497... \text{ cm}^2 \end{aligned}$$



..... 49.5 ..... cm<sup>2</sup>  
(Total for Question is 4 marks)



5. Diagram NOT accurately drawn  
The diagram shows triangle LMN.  
Calculate the length of LN.  
Give your answer correct to 3 significant figures.



$$\frac{\sin L}{12.8} = \frac{\sin 136}{15.7}$$

$$\angle L = 34.4957\dots^\circ$$

$$\angle M = 9.504\dots^\circ$$

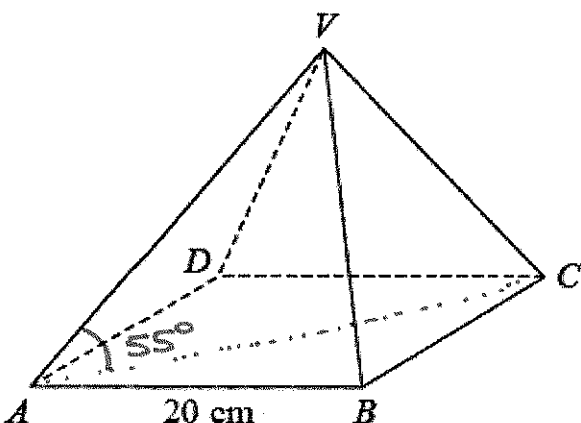
$$LN^2 = 15.7^2 + 12.8^2 - (2 \times 15.7 \times 12.8 \times \cos 9.504\dots)$$

$$LN = 3.7318\dots \text{ cm}$$

3.73

..... cm  
(Total for Question is 5 marks)

6. VABCD is a solid pyramid.  
ABCD is a square of side 20 cm.  
The angle between any sloping edge and the plane ABCD is  $55^\circ$ .  
Calculate the surface area of the pyramid.  
Give your answer correct to 2 significant figures.



$$20^2 = 400 \text{ cm}^2$$

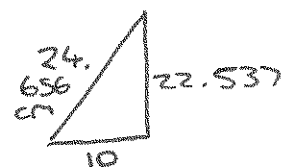
$$\frac{\sqrt{800}}{2} \div \cos 55 = 24.656\dots \text{ cm}$$

$$\sqrt{24.656\dots^2 - 10^2} = 22.537\dots \text{ cm}$$

$$\frac{20 \times 22.537\dots}{2} = 225.37\dots \text{ cm}^2$$

$$(4 \times 225.37\dots) + 400$$

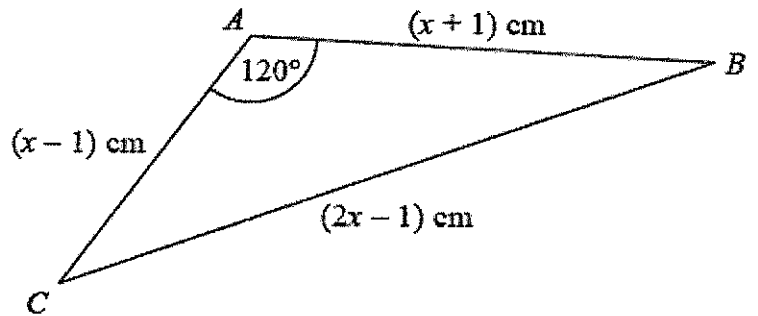
$$= 1301.4844\dots \text{ cm}^2$$



1300

..... cm<sup>2</sup>  
(Total for question = 5 marks)

7. The diagram shows triangle ABC.  
The area of triangle ABC is  $k\sqrt{3}$  cm<sup>2</sup>.  
Find the exact value of k.



$$(2x-1)^2 = (x-1)^2 + (x+1)^2 - 2(x-1)(x+1)\cos 120^\circ$$

$$4x^2 - 4x + 1 = (\cancel{x^2} - 2x + 1) + (\cancel{x^2} + 2x + 1) - (2x^2 - 2)(-\frac{1}{2})$$

$$0 = -2x^2 + 4x + 1 + x^2 + 1$$

$$0 = x^2 - 4x$$

$$x = 0 \text{ or } \boxed{4} \quad (\text{cannot equal } 0 \text{ or } AC \text{ would be } -ve)$$

$$\frac{1}{2} \times (x-1) \times (x+1) \times \sin 120$$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} \times (x^2 - 1)$$

$$= \frac{\sqrt{3}}{4} \times 15$$

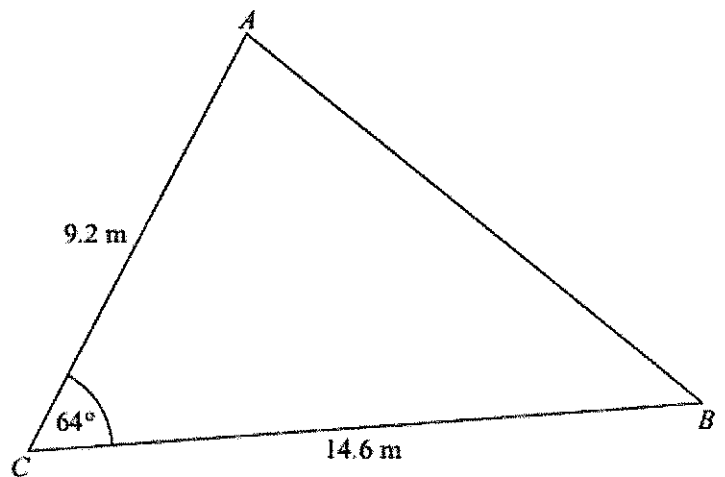
$$k = \frac{15}{4}$$

(Total for question = 7 marks)

8. Diagram NOT accurately drawn  
AC = 9.2 m  
BC = 14.6 m  
Angle ACB = 64°  
(a) Calculate the area of the triangle ABC.  
Give your answer correct to 3 significant figures.

$$\frac{1}{2} \times 9.2 \times 14.6 \times \sin 64$$

$$= 60.36300\dots \text{m}^2$$



$$\dots\dots\dots 60.4 \dots\dots\dots \text{m}^2$$

(2)

- (b) Calculate the length of AB.  
Give your answer correct to 3 significant figures.

$$AB^2 = 9.2^2 + 14.6^2 - 2(9.2)(14.6)\cos 64$$

$$AB = 13.41774\dots \text{m}$$

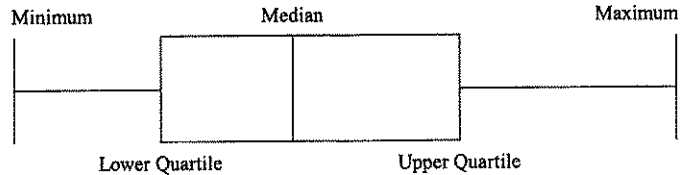
$$\dots\dots\dots 13.4 \dots\dots\dots$$

(3)  
(Total for Question is 5 marks)

## Cumulative frequency and box plots

### Things to remember:

- Use a running total – adding on to complete the cumulative frequency column;
- Plot at the end of the group;
- Join up with a smooth curve;
- To find the median find the value half way down the cumulative frequency, draw across to the line and then vertically down to find the value – always show these working lines;
- To find the interquartile range find the upper quartile and the lower quartile and subtract them.
- To draw a box plot
- When comparing box plots, use the median and the IQR and keep words consistent with the question.



### Questions:

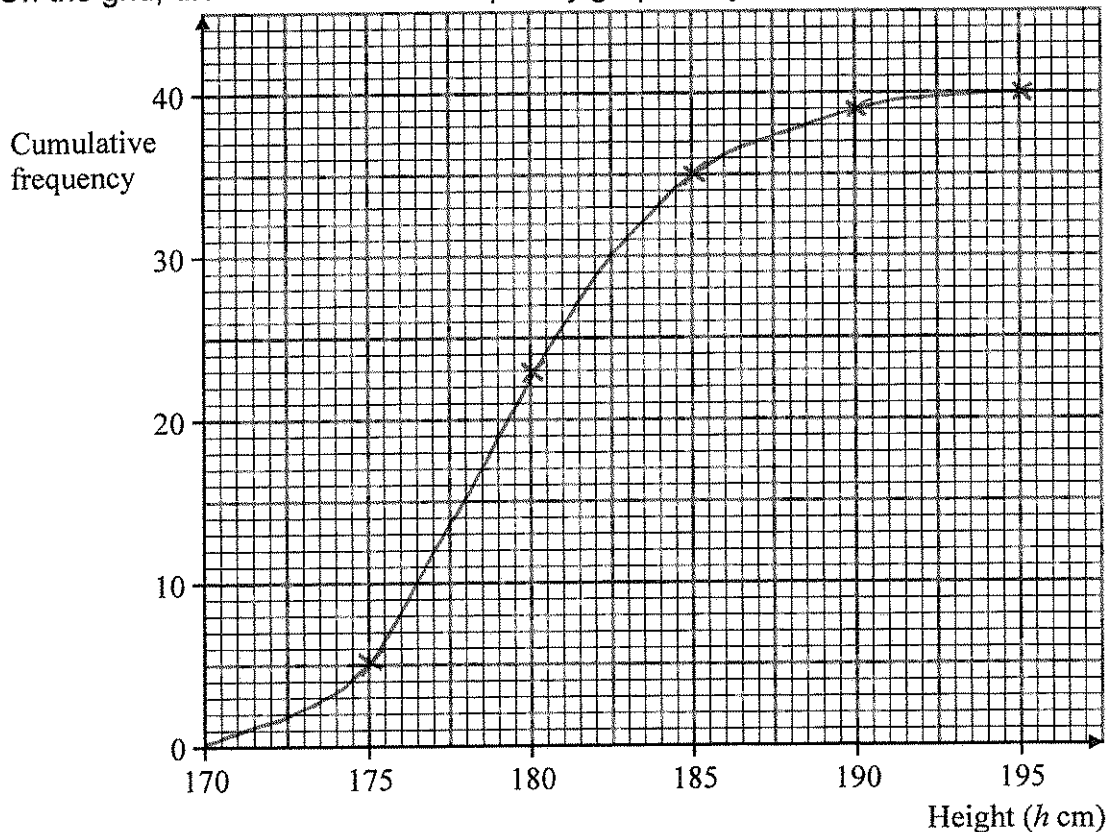
1. The table shows information about the heights of 40 bushes.

Height ( $h$ cm)	Frequency	Cumulative Frequency
$170 \leq h < 175$	5	5
$175 \leq h < 180$	18	23
$180 \leq h < 185$	12	35
$185 \leq h < 190$	4	39
$190 \leq h < 195$	1	40

(a) Complete the cumulative frequency table above.

(1)

(b) On the grid, draw a cumulative frequency graph for your table.



(2)

(Total 3 marks)

2. The table gives information about the ages of 160 employees of an IT company.

Age ( $A$ ) in years	Frequency	Cumulative Frequency
$15 < A \leq 25$	44	44
$25 < A \leq 35$	56	100
$35 < A \leq 45$	34	134
$45 < A \leq 55$	19	153
$55 < A \leq 65$	7	160

(a) Write down the modal class interval.

$25 < A \leq 35$

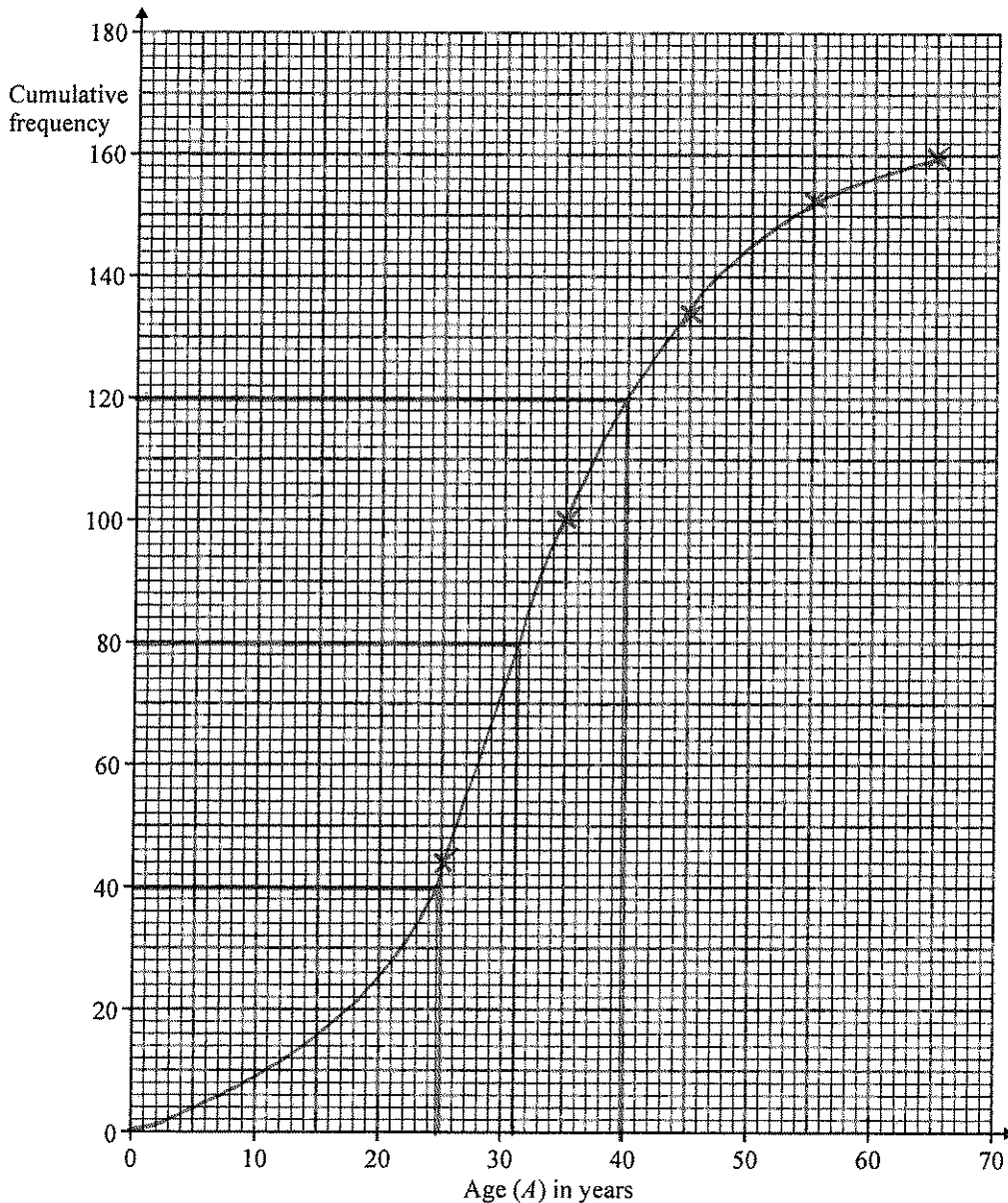
(1)

(b) Complete the cumulative frequency table.

(1)

(c) On the grid below, draw a cumulative frequency graph for your table.

(2)



(d) Use your graph to find an estimate for

(i) the median age of the employees,

..... 31 ..... years

(ii) the interquartile range of the ages of the employees.

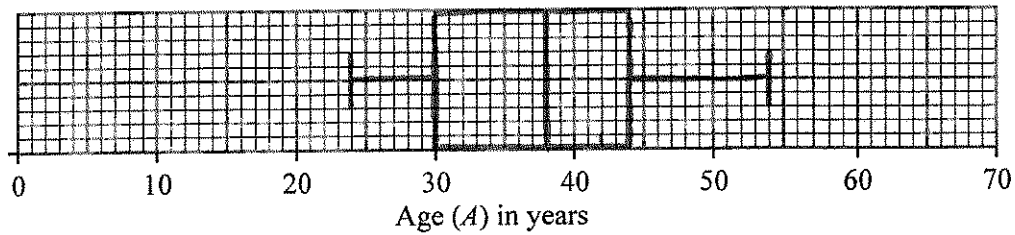
..... 40 - 25 ..... 15 ..... years

(3)

Another IT company has 80 employees. The age of the youngest employee is 24 years. The age of the oldest employee is 54 years. The median age is 38 years. The lower quartile age is 30 years. The upper quartile age is 44 years.

- (e) On the grid below, draw a box plot to show information about the ages of the employees.

(2)



(Total 9 marks)

3. A company tested 100 batteries. The table shows information about the number of hours that the batteries lasted.

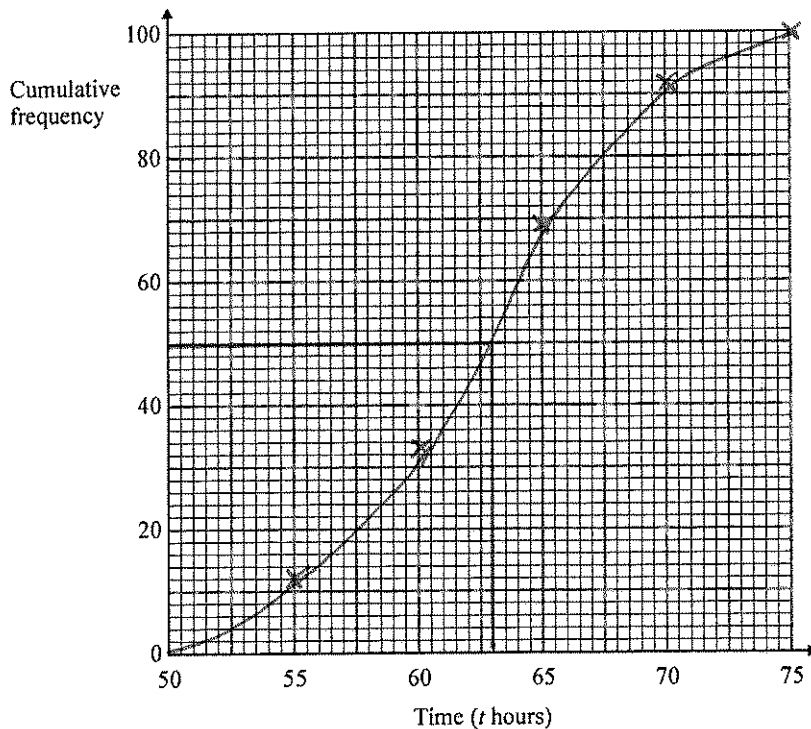
Time ( $t$ hours)	Frequency	Cumulative Frequency
$50 \leq t < 55$	12	12
$55 \leq t < 60$	21	33
$60 \leq t < 65$	36	69
$65 \leq t < 70$	23	92
$70 \leq t < 75$	8	100

- (a) Complete the cumulative frequency table for this information.

(1)

- (b) On the grid, draw a cumulative frequency graph for your completed table.

(2)



- (c) Use your completed graph to find an estimate for the median time. You must state the units of your answer.

..... 63 hours .....

(2)

(Total 5 marks)

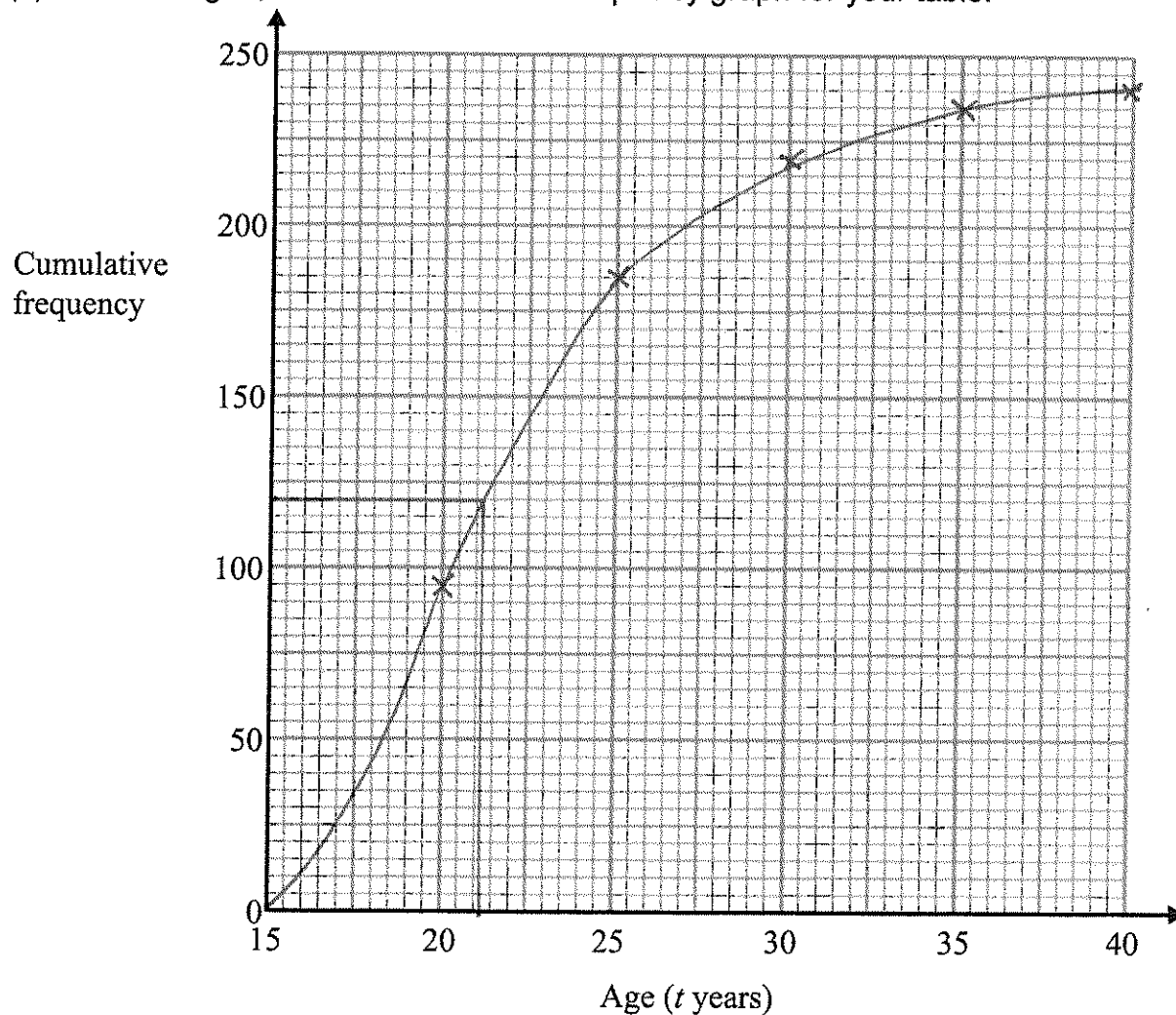
4. The table shows information about the ages of the 240 people at a club.

Age ( $t$ years)	Frequency	Cumulative Frequency
$15 \leq t < 20$	95	95
$20 \leq t < 25$	90	185
$25 \leq t < 30$	35	220
$30 \leq t < 35$	15	235
$35 \leq t < 40$	5	240

(a) Complete the cumulative frequency table.

(1)

(b) On the grid, draw the cumulative frequency graph for your table.



(c) Use your graph to find an estimate for the median age of the people.

(2)

..... 21 ..... years

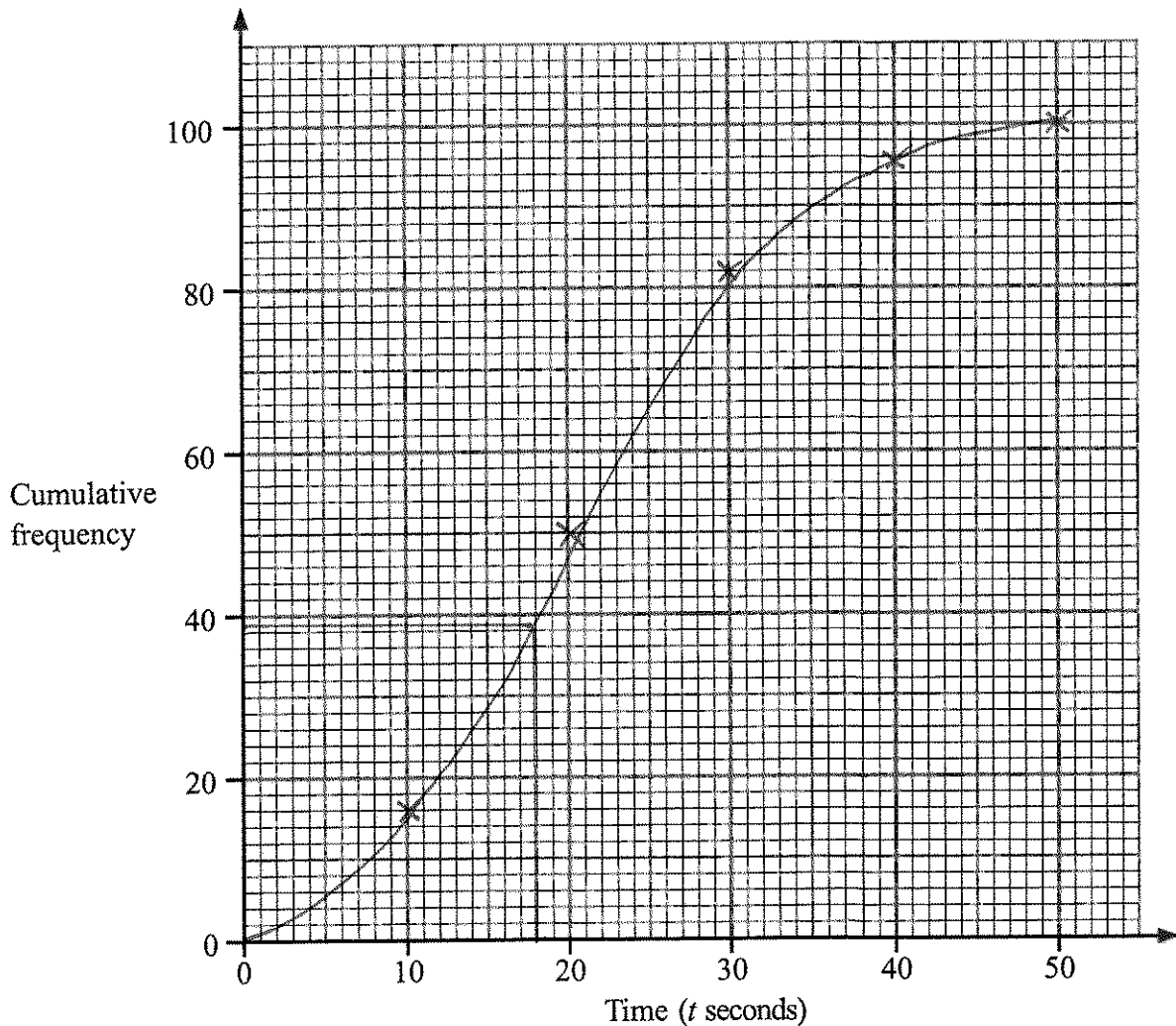
(1)

(Total 4 marks)

5. An operator took 100 calls at a call centre. The table gives information about the time ( $t$  seconds) it took the operator to answer each call.

Time ( $t$ seconds)	Frequency	Cumulative Frequency
$0 < t \leq 10$	16	16
$10 < t \leq 20$	34	50
$20 < t \leq 30$	32	82
$30 < t \leq 40$	14	96
$40 < t \leq 50$	4	100

- (a) Complete the cumulative frequency table. (1)
- (b) On the grid, draw a cumulative frequency graph for your table. (2)



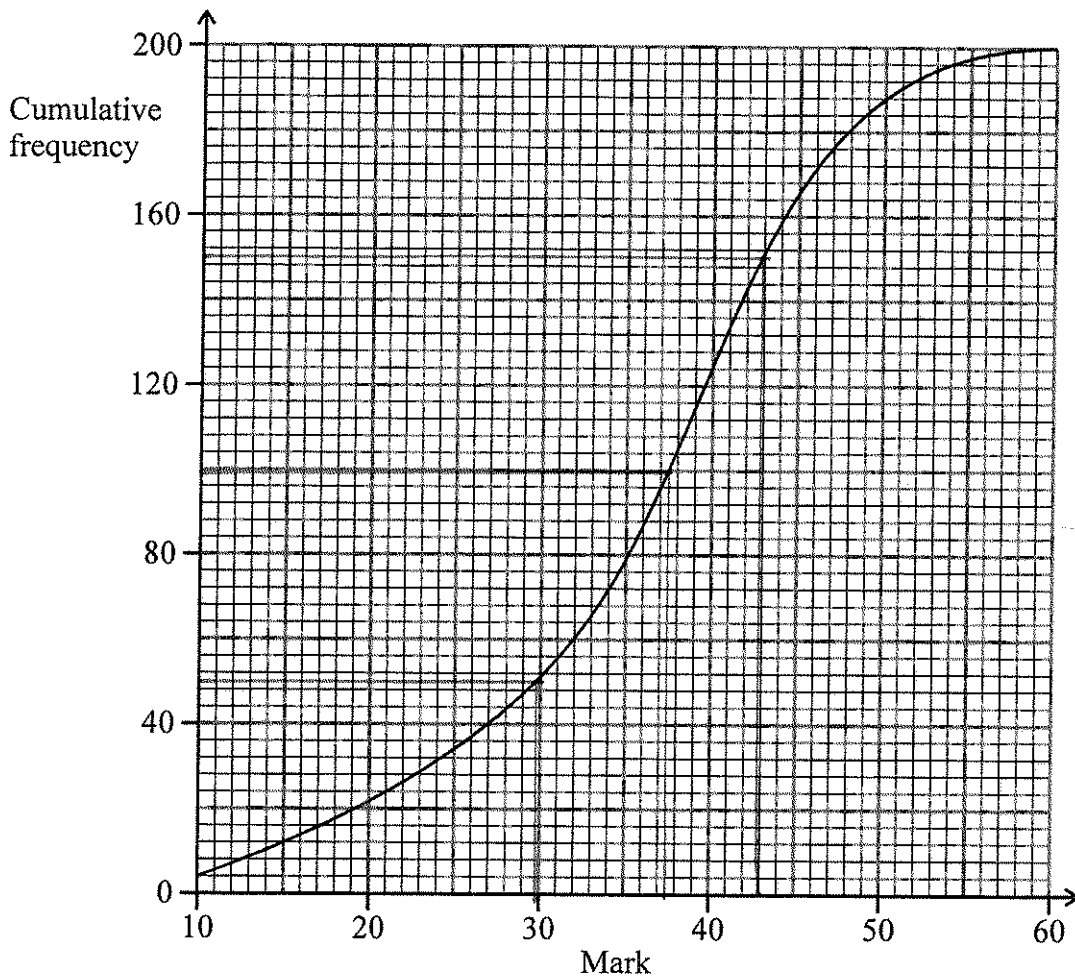
- (c) Use your graph to find an estimate for the number of calls the operator took more than 18 seconds to answer.

$$100 - 39$$

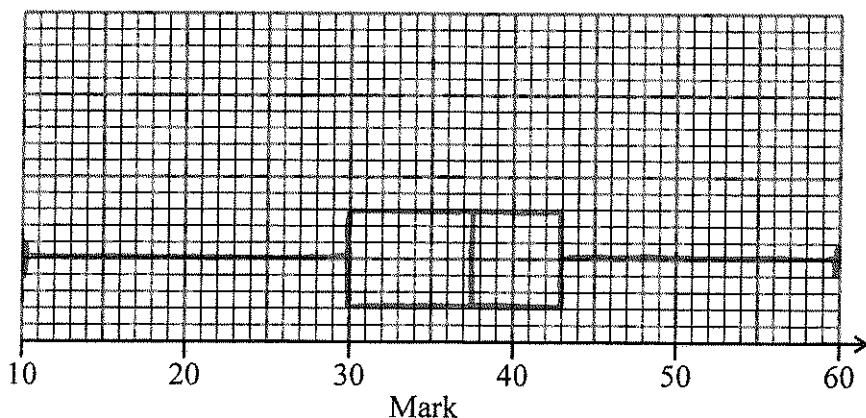
..... 61 .....

(2)  
(Total 5 marks)

6. 200 students took a test. The cumulative frequency graph gives information about their marks.



The lowest mark scored in the test was 10.  
 The highest mark scored in the test was 60.  
 Use this information and the cumulative frequency graph to draw a box plot showing information about the students' marks.



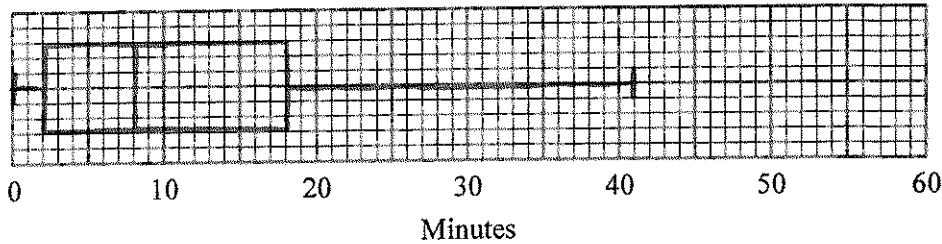
(Total 3 marks)



7. On Friday, Peter went to the airport. He recorded the number of minutes that each plane was delayed. He used his results to work out the information in this table.

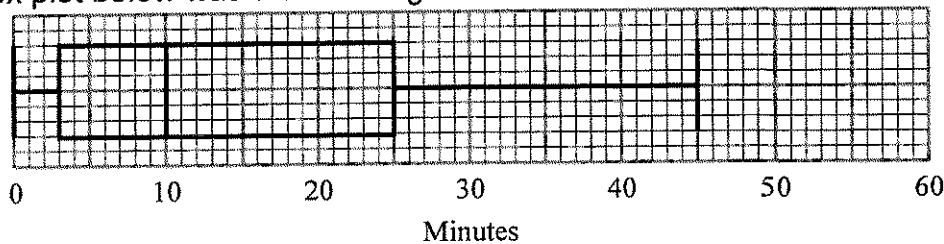
	Minutes
Shortest delay	0
Lower quartile	2
Median	8
Upper quartile	18
Longest delay	41

- (a) On the grid, draw a box plot to show the information in the table.



(2)

Peter also went to the airport on Saturday. He recorded the number of minutes that each plane was delayed. The box plot below was drawn using this information.



- (b) Make two comparisons between the distributions of plane delays on Friday and on Saturday.

On average, planes were later on Saturday.  
 The planes were of a more consistent delay on Friday (smaller IQR)

(2)

(Total 4 marks)

## Histograms

### Things to remember:

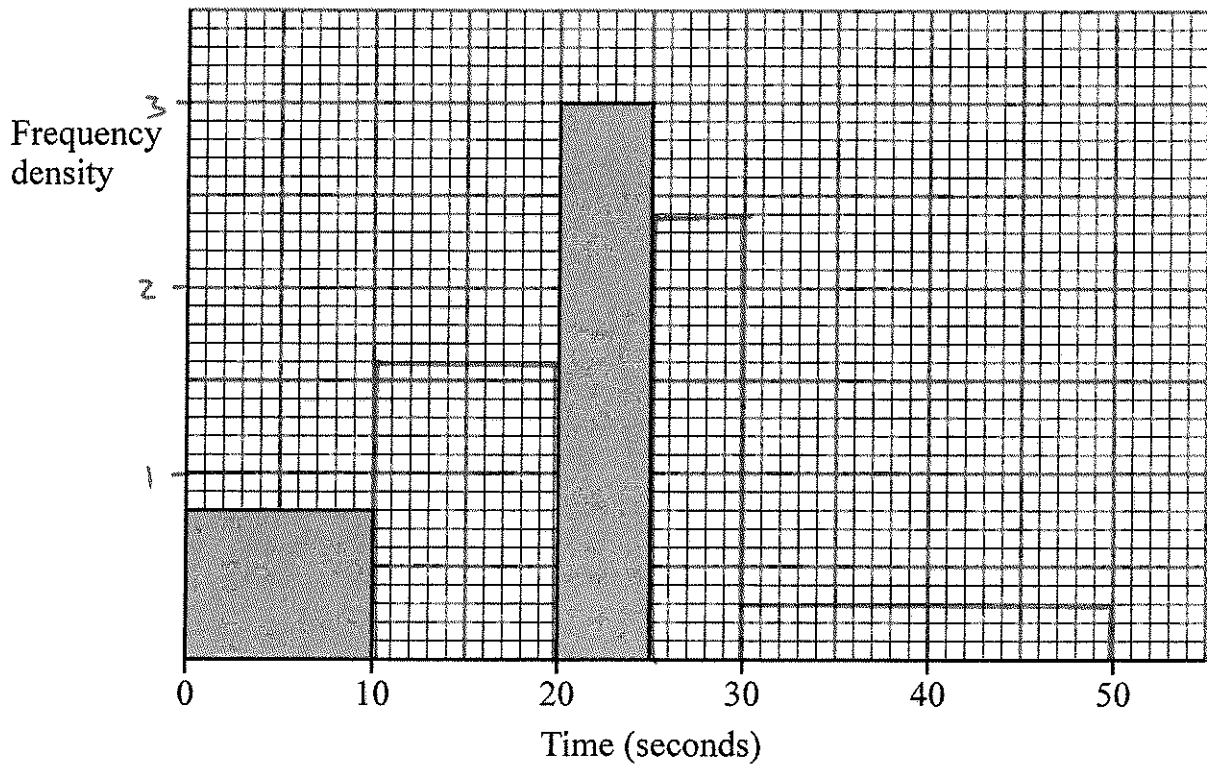
- Frequency = Frequency Density  $\times$  Class Width;
- The y-axis will always be labelled "frequency density";
- The x-axis will have a continuous scale.

### Questions:

1. One Monday, Victoria measured the time, in seconds, that individual birds spent on her bird table. She used this information to complete the frequency table.

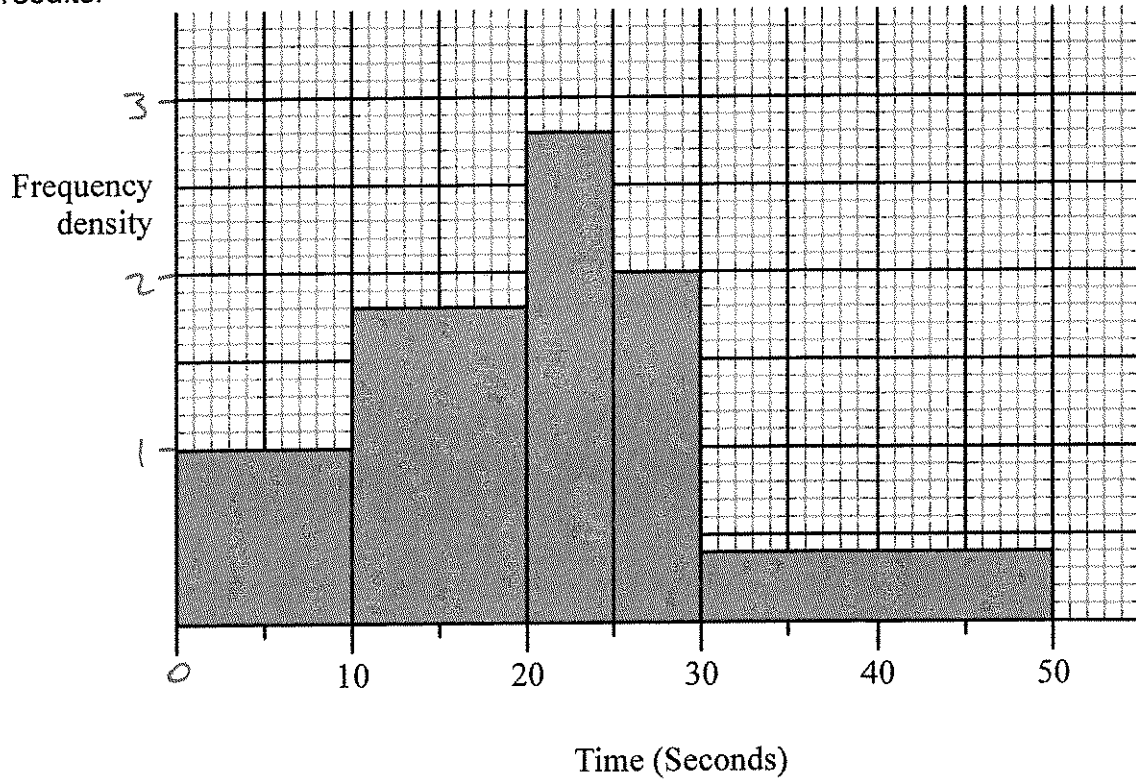
Time ( $t$ seconds)	Frequency	FD
$0 < t \leq 10$	8	0.8
$10 < t \leq 20$	16	1.6
$20 < t \leq 25$	15	3
$25 < t \leq 30$	12	2.4
$30 < t \leq 50$	6	0.3

- (a) Use the table to complete the histogram.



(3)

On Tuesday she conducted a similar survey and drew the following histogram from her results.

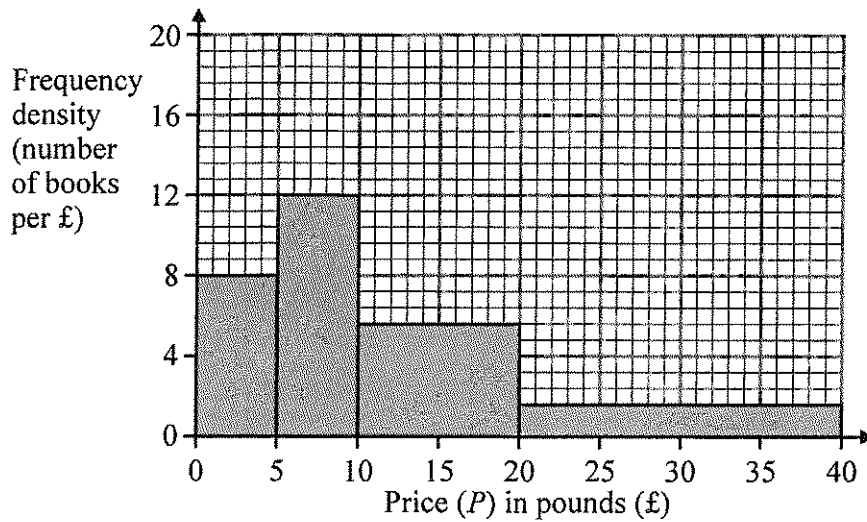


(b) Use the histogram for Tuesday to complete the table.

Time ( $t$ seconds)	Frequency	FD
$0 < t \leq 10$	10	1
$10 < t \leq 20$	18	1.8
$20 < t \leq 25$	14	2.8
$25 < t \leq 30$	10	2
$30 < t \leq 50$	8	0.4

(2)  
(Total 5 marks)

2. This histogram gives information about the books sold in a bookshop one Saturday.



- (a) Use the histogram to complete the table.

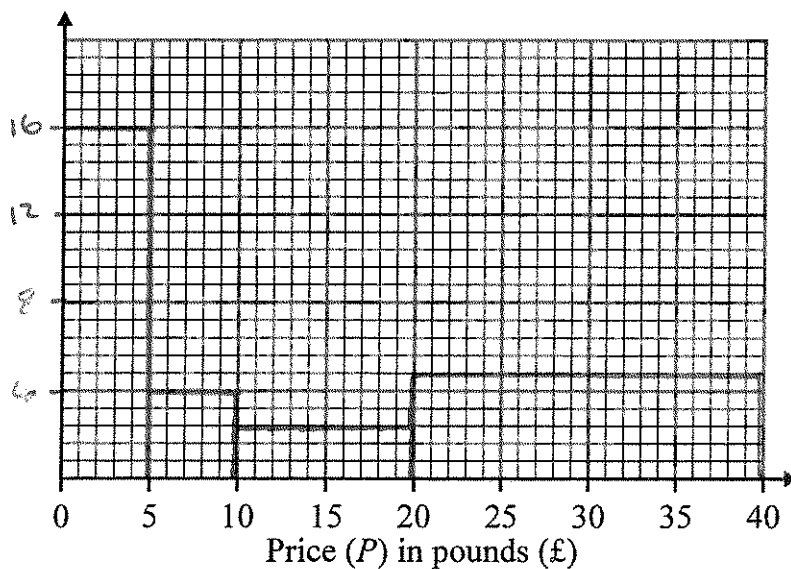
Price ( $P$ ) in pounds (£)	Frequency
$0 < P \leq 5$	40
$5 < P \leq 10$	60
$10 < P \leq 20$	56
$20 < P \leq 40$	32

(2)

The frequency table below gives information about the books sold in a second bookshop on the same Saturday.

Price ( $P$ ) in pounds (£)	Frequency	FD
$0 < P \leq 5$	80	16
$5 < P \leq 10$	20	4
$10 < P \leq 20$	24	2.4
$20 < P \leq 40$	96	4.8

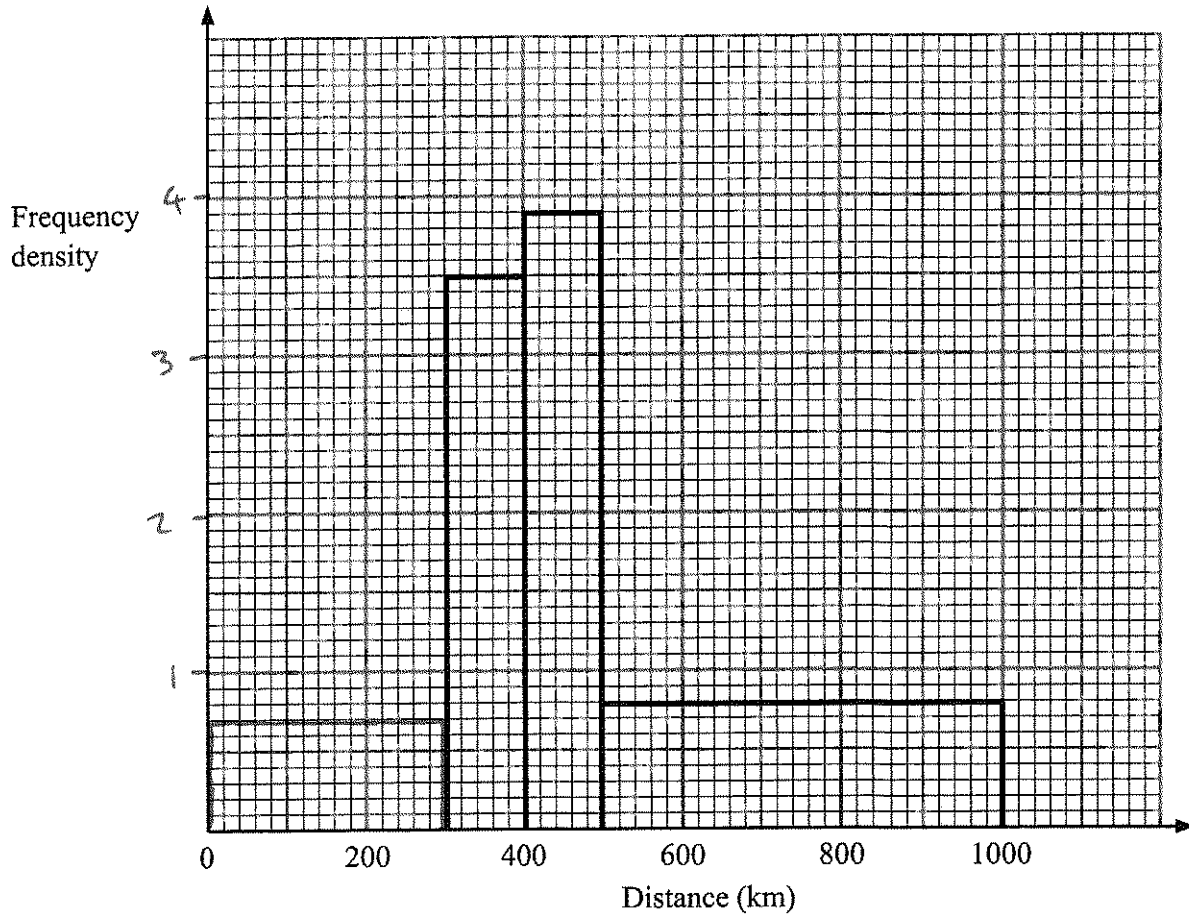
- (b) On the grid below, draw a histogram to represent the information about the books sold in the second bookshop.



(3)

(Total 5 marks)

3. The incomplete table and histogram give some information about the distances walked by some students in a school in one year.



- (a) Use the information in the histogram to complete the frequency table.

Distance ( $d$ ) in km	Frequency	FD
$0 < d \leq 300$	210	0.7
$300 < d \leq 400$	350	3.5
$400 < d \leq 500$	390	3.9
$500 < d \leq 1000$	400	0.8

(2)

- (b) Use the information in the table to complete the histogram.

(1)

(Total 3 marks)

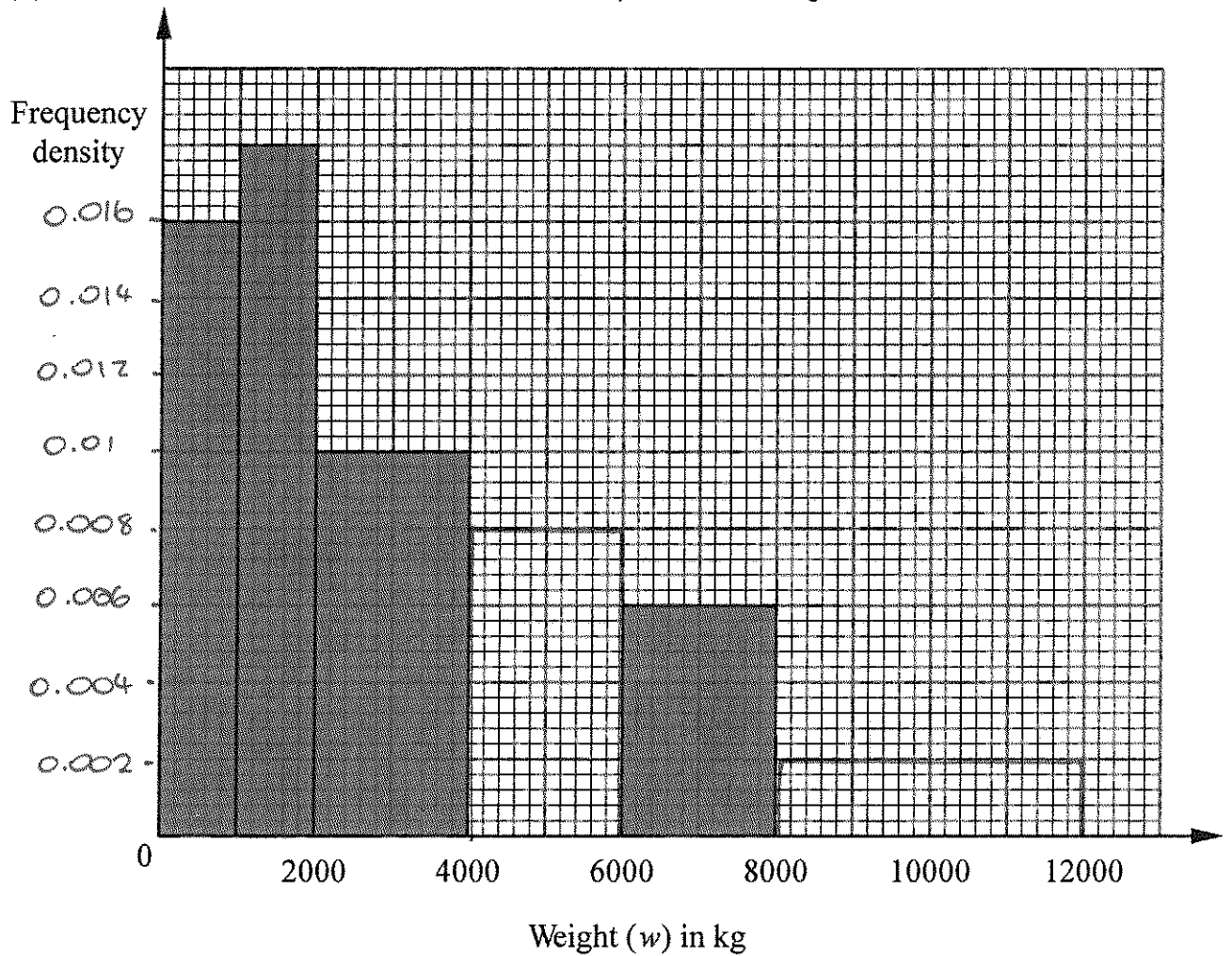
4. The incomplete histogram and table show information about the weights of some containers.

Weight ( $w$ ) in kg	Frequency	FD
$0 < w \leq 1000$	16	0.016
$1000 < w \leq 2000$	18	0.018
$2000 < w \leq 4000$	20	0.01
$4000 < w \leq 6000$	16	0.008
$6000 < w \leq 8000$	12	0.006
$8000 < w \leq 12000$	8	

(a) Use the information in the histogram to complete the table.

(2)

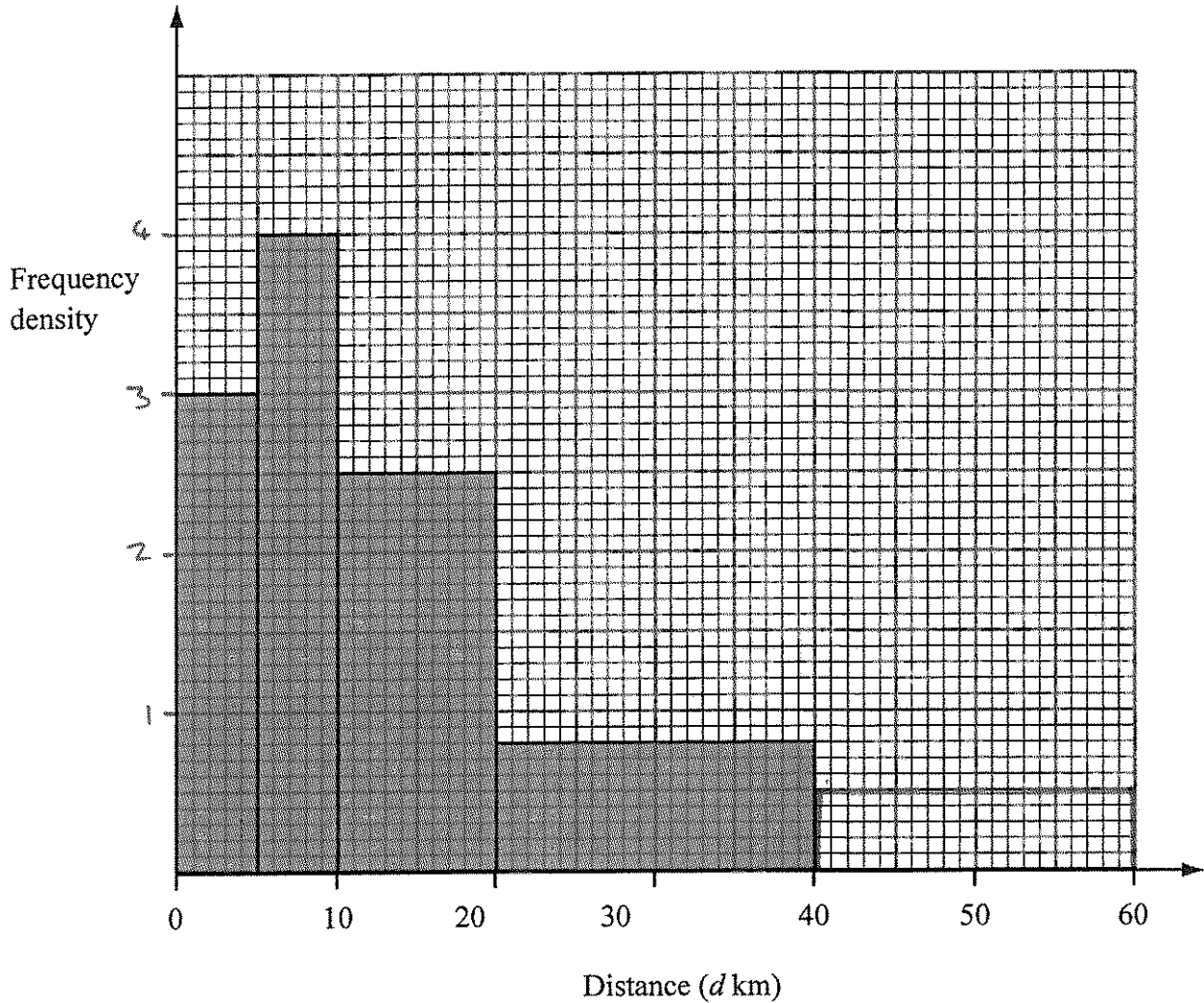
(b) Use the information in the table to complete the histogram.



(2)

(Total 4 marks)

5. The incomplete histogram and table give some information about the distances some teachers travel to school.



- (a) Use the information in the histogram to complete the frequency table.

Distance ( $d$ km)	Frequency	FD
$0 < d \leq 5$	15	3
$5 < d \leq 10$	20	4
$10 < d \leq 20$	25	2.5
$20 < d \leq 40$	16	0.8
$40 < d \leq 60$	10	0.5

- (b) Use the information in the table to complete the histogram.

(2)  
(1)  
(Total 3 marks)

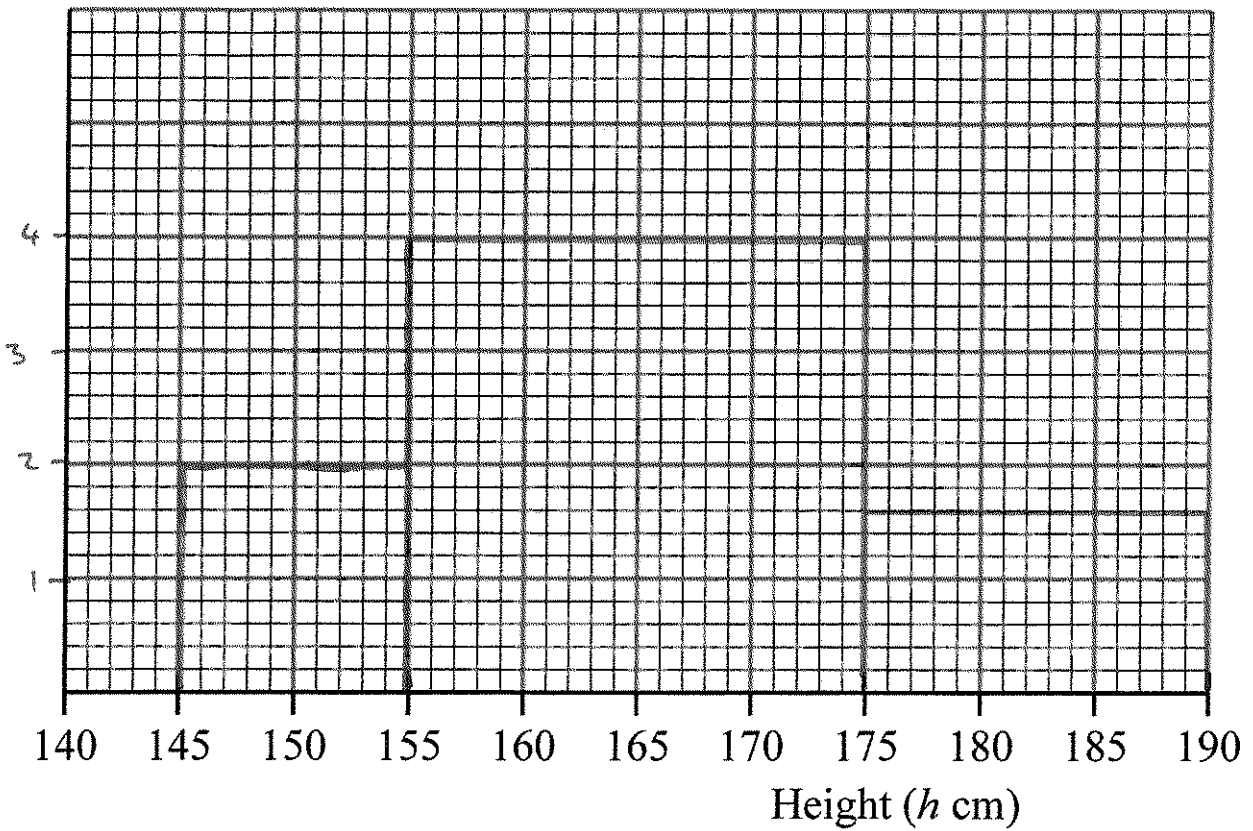
6. The table gives information about the heights, in centimetres, of some 15 year old students.

Height ( $h$ cm)	$145 < h \leq 155$	$155 < h \leq 175$	$175 < h \leq 190$
Frequency	10	80	24

$$\frac{24}{15} = \frac{8}{5}$$

FD                      2                      4                      1.6

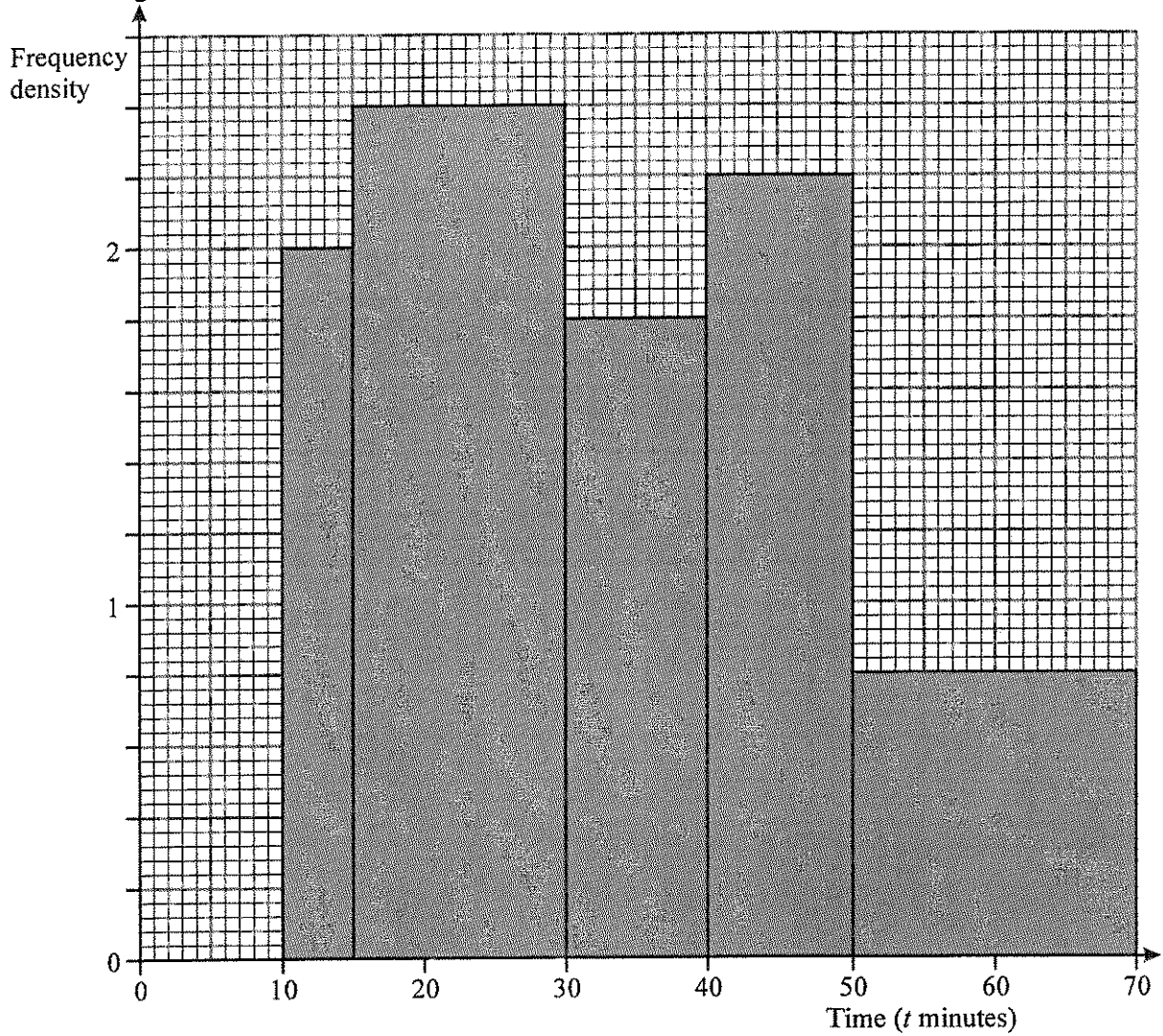
Use the table to draw a histogram.



(Total 3 marks)



7. A teacher asked some year 10 students how long they spent doing homework each night. The histogram was drawn from this information.



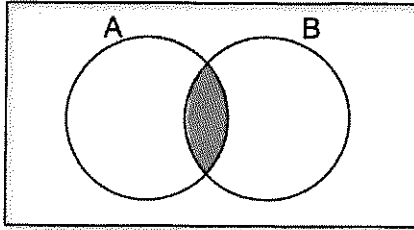
Use the histogram to complete the table.

Time ( $t$ minutes)	Frequency	FD
$10 \leq t < 15$	10	2
$15 \leq t < 30$	36	2.4
$30 \leq t < 40$	18	1.8
$40 \leq t < 50$	22	2.2
$50 \leq t < 70$	16	0.8

(Total 2 marks)

## Set Theory

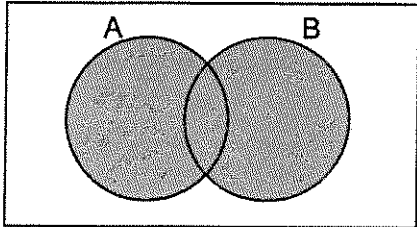
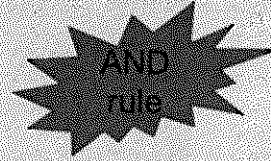
Things to remember:



The **intersection** is where two sets overlap.

$$A \cap B$$

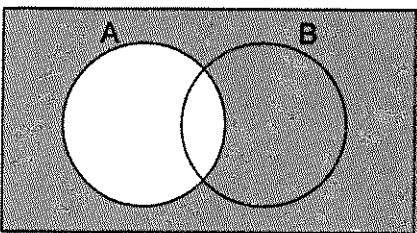
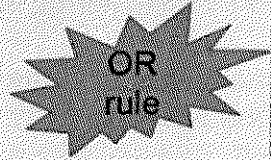
This means **A and B**.



If you put two sets together, you get the **union**.

$$A \cup B$$

This means **A or B**.



The **complement of A** is the region that is not A.

$$A'$$

This means **not A**.

Questions:

1.

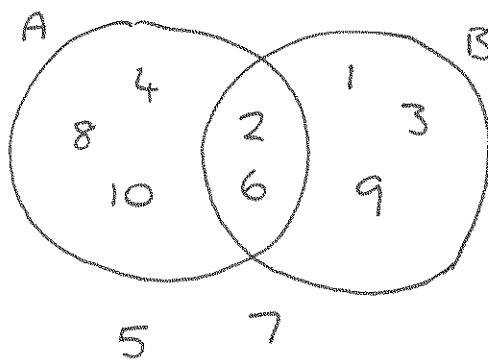
$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{\text{multiples of 2}\}$$

$$A \cap B = \{2, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 6, 8, 9, 10\}$$

Draw a Venn diagram for this information.



(Total for question is 4 marks)

2. Here is a Venn diagram.

(a) Write down the numbers that are in set

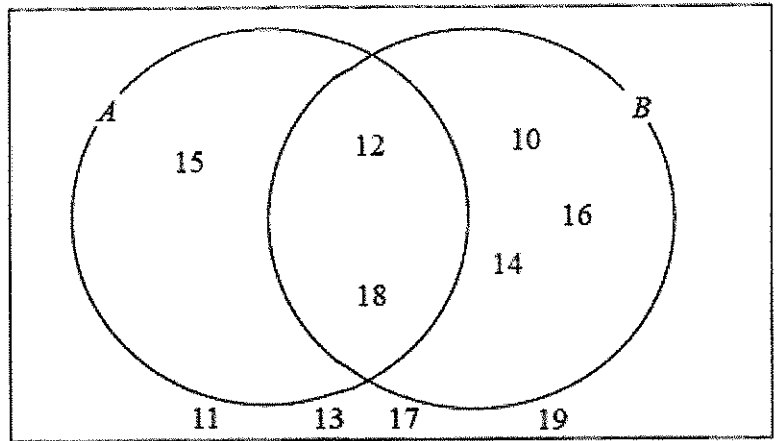
(i)  $A \cup B$

*15, 12, 18, 14, 10, 16*

(ii)  $A \cap B$

*12, 18*

(2)



One of the numbers in the diagram is chosen at random.

(b) Find the probability that the number is in set  $A'$

$$P(A') = \frac{7}{10}$$

$$\frac{7}{10}$$

(2)

(Total for question = 4 marks)

3. Here is a Venn diagram.

(a) Write down the numbers that are in set

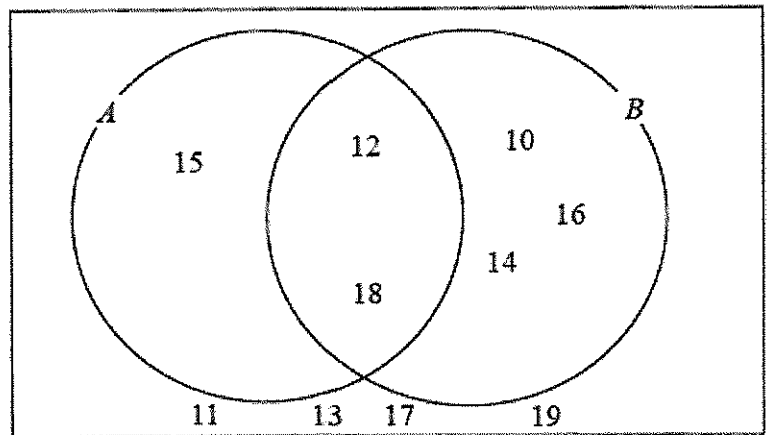
(i)  $A \cup B$

.....

(ii)  $A \cap B$

.....

(2)



One of the numbers in the diagram is chosen at random.

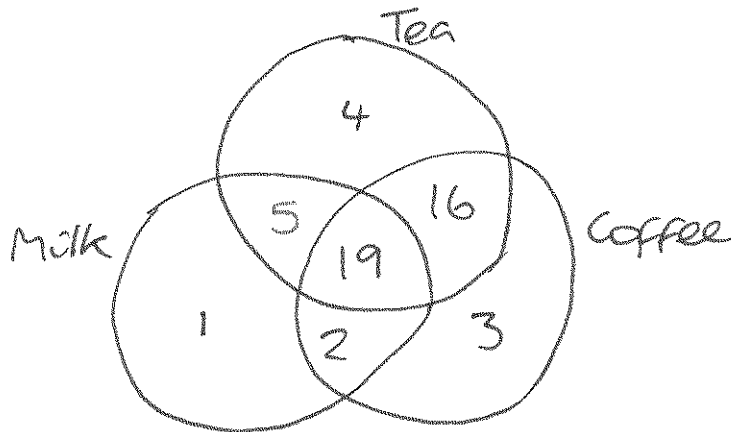
(b) Find the probability that the number is in set  $A'$

*Same as above - sorry!*

(2)

(Total for question = 4 marks)

4. Sami asked 50 people which drinks they liked from tea, coffee and milk.  
 All 50 people like at least one of the drinks  
~~19~~ people like all three drinks.  
~~16~~ people like tea and coffee but do not like milk.  
~~21~~ people like coffee and milk.  
~~24~~ people like tea and milk.  
~~40~~ people like coffee.  
 1 person likes only milk.  
 Sami selects at random one of the 50 people.  
 (a) Work out the probability that this person likes tea.



$$\frac{44}{50}$$

- (b) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink. (4)

$$\frac{21}{44}$$

(2)  
 (Total for question = 6 marks)

## Proportion

### Things to remember:

- Start by checking the question for squares, cubes and roots;
- "x is directly proportional to y" looks like  $x \propto y$  or  $x = ky$
- "x is inversely proportional to y" looks like  $x \propto \frac{1}{y}$  or  $x = \frac{k}{y}$

### Questions:

1. The shutter speed,  $S$ , of a camera varies inversely as the square of the aperture setting,  $f$ .  
When  $f = 8$ ,  $S = 125$

(a) Find a formula for  $S$  in terms of  $f$ .

$$S \propto \frac{1}{f^2}$$

$$125 = \frac{k}{8^2}$$

$$k = 8000$$

$$S = \frac{8000}{f^2} \dots\dots\dots (3)$$

(b) Hence, or otherwise, calculate the value of  $S$  when  $f = 4$

$$S = \frac{8000}{4}$$

$$S = \dots\dots\dots 2000 \dots\dots\dots (1)$$

(Total 4 marks)

2. In a factory, chemical reactions are carried out in spherical containers.  
The time,  $T$  minutes, the chemical reaction takes is directly proportional to the square of the radius,  $R$  cm, of the spherical container.  
When  $R = 120$ ,  $T = 32$   
Find the value of  $T$  when  $R = 150$

$$T \propto r^2$$

$$32 = k \times 120^2$$

$$\frac{1}{450} = k$$

$$T = \frac{R^2}{450}$$

$$T = \dots\dots\dots 50 \dots\dots\dots$$

(Total 4 marks)

3.  $d$  is directly proportional to the square of  $t$ .

$d = 80$  when  $t = 4$

(a) Express  $d$  in terms of  $t$ .

$$d \propto t^2$$
$$80 = k \times 4^2$$
$$k = 5$$

$$d = 5t^2$$

.....

(3)

(b) Work out the value of  $d$  when  $t = 7$

$$d = 5 \times 7^2$$

$$d = \dots 245$$

.....

(1)

(c) Work out the positive value of  $t$  when  $d = 45$

$$45 = 5t^2$$
$$9 = t^2$$

$$t = \dots 3$$

.....

(2)

(Total 6 marks)

4. The distance,  $D$ , travelled by a particle is directly proportional to the square of the time,  $t$ , taken. When  $t = 40$ ,  $D = 30$

(a) Find a formula for  $D$  in terms of  $t$ .

$$D \propto t^2$$
$$30 = k \times 40^2$$
$$k = 0.01875$$

$$D = \dots D = 0.01875t^2$$

.....

(3)

(b) Calculate the value of  $D$  when  $t = 64$

$$D = 0.01875 \times 64^2$$

$$76.8$$

.....

(1)

(c) Calculate the value of  $t$  when  $D = 12$   
Give your answer correct to 3 significant figures.

$$12 = 0.01875t^2$$
$$t = 25.29822\dots$$

$$25.3$$

.....

(2)

(Total 6 marks)

5. The time,  $T$  seconds, it takes a water heater to boil some water is directly proportional to the mass of water,  $m$  kg, in the water heater. When  $m = 250$ ,  $T = 600$

(a) Find  $T$  when  $m = 400$

$$T \propto m$$

$$600 = k \times 250$$

$$k = 2.4$$

$$T = 2.4m$$

$$T = 400 \times 2.4$$

$$T = \dots \dots \dots 960 \dots \dots \dots (3)$$

The time,  $T$  seconds, it takes a water heater to boil a constant mass of water is inversely proportional to the power,  $P$  watts, of the water heater.

When  $P = 1400$ ,  $T = 360$

(b) Find the value of  $T$  when  $P = 900$

$$T \propto \frac{1}{P}$$

$$360 = \frac{k}{1400}$$

$$k = 504000$$

$$T = \frac{504000}{P}$$

$$T = \frac{504000}{900}$$

$$T = \dots \dots \dots 560 \dots \dots \dots (3)$$

(Total 6 marks)

6. A ball falls vertically after being dropped.  
The ball falls a distance  $d$  metres in a time of  $t$  seconds.  
 $d$  is directly proportional to the square of  $t$ .  
The ball falls 20 metres in a time of 2 seconds.

(a) Find a formula for  $d$  in terms of  $t$ .

$$d \propto t^2$$

$$20 = k \times 2^2$$

$$k = 5$$

$$d = \dots \dots \dots d = 5t^2 \dots \dots \dots (3)$$

(b) Calculate the distance the ball falls in 3 seconds.

$$d = 5 \times 3^2$$

$$\dots \dots \dots 45 \dots \dots \dots \text{m} \dots \dots \dots (1)$$

(c) Calculate the time the ball takes to fall 605 m.

$$605 = 5t^2$$

$$121 = t^2$$

$$\dots \dots \dots 11 \dots \dots \dots \text{seconds} \dots \dots \dots (3)$$

(Total 7 marks)

7. In a spring, the tension ( $T$  newtons) is directly proportional to its extension ( $x$  cm). When the tension is 150 newtons, the extension is 6 cm.

(a) Find a formula for  $T$  in terms of  $x$ .

$$T \propto x$$

$$150 = k \times 6$$

$$k = 25$$

$$T = 25x$$

$$T = \dots 25x \dots \dots \dots$$

(3)

(b) Calculate the tension, in newtons, when the extension is 15 cm.

$$25 \times 15$$

$$\dots 375 \dots \dots \dots \text{newtons}$$

(1)

(c) Calculate the extension, in cm, when the tension is 600 newtons.

$$600 = 25x$$

$$\dots 24 \dots \dots \dots \text{cm}$$

(1)

(Total 5 marks)

8.  $f$  is inversely proportional to  $d$ .  
When  $d = 50$ ,  $f = 256$   
Find the value of  $f$  when  $d = 80$

$$f \propto \frac{1}{d}$$

$$256 = \frac{k}{50}$$

$$k = 12800$$

$$f = \frac{12800}{d}$$

$$f = \frac{12800}{80}$$

$$f = \dots 160 \dots \dots \dots$$

(Total 3 marks)



## Percentages – compound interest

### Things to remember:

- New amount = original amount  $\times$  multiplier<sup>n</sup>

Number of years

### Questions:

1. Henry invests £4500 at a compound interest rate of 5% per annum. At the end of  $n$  complete years the investment has grown to £5469.78. Find the value of  $n$ .

$$4500 \times 1.05^n$$

$$n = 4$$

(Total 2 marks)

2. Bill buys a new machine. The value of the machine depreciates by 20% each year.  
(a) Bill says 'after 5 years the machine will have no value'. Bill is **wrong**. Explain why.

The value of the 20% will change as the value of the machine changes year on year.

(1)

Bill wants to work out the value of the machine after 2 years.

- (b) By what single decimal number should Bill multiply the value of the machine when new?

$$0.8 \times 0.8$$

$$0.64$$

(2)

(Total 3 marks)

3. Gwen bought a new car. Each year, the value of her car depreciated by 9%. Calculate the number of years after which the value of her car was 47% of its value when new.

$$0.91^n$$

$$n = 8$$

(Total 3 marks)

4. The value of a car depreciates by 35% each year. At the end of 2007 the value of the car was £5460. Work out the value of the car at the end of 2006.

$$5460 \div 0.65$$

$$£ 8400$$

(Total 3 marks)

5. Toby invested £4500 for 2 years in a savings account.  
He was paid 4% per annum compound interest.  
(a) How much did Toby have in his savings account after 2 years?

$$4500 \times 1.04^2$$

£ ..... 4867.20 ..... (3)

- Jaspir invested £2400 for  $n$  years in a savings account.  
He was paid 7.5% per annum compound interest.  
At the end of the  $n$  years he had £3445.51 in the savings account.  
(a) Work out the value of  $n$ .

$$2400 \times 1.075^n$$

.....  $n=5$  ..... (2)  
(Total 5 marks)

6. Mario invests £2000 for 3 years at 5% per annum **compound** interest.  
Calculate the value of the investment at the end of 3 years.

$$2000 \times 1.05^3$$

£ ..... 2315.25 ..... (Total 3 marks)

7. Toby invested £4500 for 2 years in a savings account.  
He was paid 4% per annum compound interest.  
How much did Toby have in his savings account after 2 years?

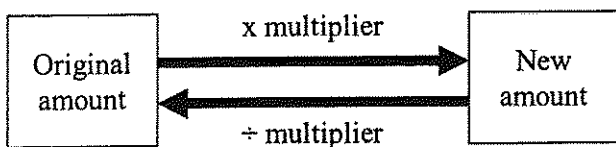
$$4500 \times 1.04^2$$

£ ..... 4867.20 ..... (Total 3 marks)

## Percentages – reverse

### Things to remember:

- Work out what the multiplier would have been;



### Questions:

1. Loft insulation reduces annual heating costs by 20%.  
**After** he insulated his loft, Curtley's annual heating cost was £520.  
Work out Curtley's annual heating cost would have been, if he had not insulated his loft.

$$520 \div 0.8$$

£ ..... 650 .....  
(Total 3 marks)

2. In a sale, normal prices are reduced by 20%.  
**SALE - 20% OFF**  
Andrew bought a saddle for his horse in the sale.  
The sale price of the saddle was £220.  
Calculate the normal price of the saddle.

$$220 \div 0.8$$

£ ..... 275 .....  
(Total 3 marks)

3. Hajra's weekly pay this year is £240  
This is 20% more than her weekly pay last year.  
Bill says 'This means Hajra's weekly pay last year was £192'.  
Bill is wrong,

- (a) Explain why.

..... Bill took the £240 to be 100% rather .....  
..... than 120% .....  
(1)

- (b) Work out Hajra's weekly pay last year.

$$240 \div 1.2$$

£ ..... 200 .....  
(2)  
(Total 3 marks)

4. The price of all rail season tickets to London increased by 4%.  
(a) The price of a rail season ticket from Cambridge to London increased by £121.60  
Work out the price before this increase.

$$\begin{aligned} \pounds 121.60 &= 4\% \\ &\times 25. \end{aligned}$$

£ ..... 3040 ..... (2)

- (b) After the increase, the price of a rail season ticket from Brighton to London was £2828.80  
Work out the price before this increase.

$$\pounds 2828.80 \div 1.04$$

£ ..... 2720 ..... (3)  
(Total 5 marks)

5. In a sale, normal prices are reduced by 25%.  
The sale price of a saw is £12.75  
Calculate the normal price of the saw.

$$12.75 \div 0.75$$

£ ..... 17 ..... (Total 3 marks)

6. In a sale, normal prices are reduced by 12%.  
The sale price of a DVD player is £242.  
Work out the normal price of the DVD player.

$$242 \div 0.88$$

£ ..... 275 ..... (Total 3 marks)

7. A garage sells cars.  
It offers a discount of 20% off the normal price for cash.  
Dave pays £5200 cash for a car.  
Calculate the normal price of the car.

$$5200 \div 0.8$$

£ ..... 6500 ..... (Total 3 marks)